Integration of Graphing Calculators into the Pre-university Mathematics Curriculum

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Abstract

In Singapore the use of graphing calculators without a built-in computer algebra system in public examinations was first permitted in the year 2002 for Further Mathematics, a subject offered at the pre-university level. Following the inception in 2006 of the revised mathematics syllabus and curriculum, not only is the use of graphing calculators incorporated in assessment for all pre-university level mathematics subjects, all main stream schools offering mathematics subjects at the pre-university level are now expected to fully integrate the graphing calculator into the mathematics classroom. As a result, teachers in these schools are required to be proficient in utilising the graphing calculator and adept at facilitating students' usage of the graphing calculator so as to meet the new assessment requirements. This paper delineates the rationale for the integration of the graphing calculator into the new curriculum and discusses the impact of the graphing calculator on the Further Mathematics curriculum. Issues pertaining to professional development and training of teachers in mastering the use of a graphing calculator and the impact of the graphing calculator on assessment in the new curriculum are also discussed.

Introduction

Since the debut of the electronic calculator more than 40 years ago, there has been increased focus on the use of handheld technology in enhancing teaching and learning of mathematics (Pomerantz, 1997). In Singapore, scientific calculators were first integrated into the secondary school curricula in early 1980's and the use of graphing calculators (GCs) without a built-in computer algebra system (CAS), or non-CAS GCs, in public examinations was first permitted in the year 2002 for Further Mathematics, a subject offered at the advanced level (i.e. pre-university level) for students with a keen interest in, and have an aptitude for, mathematics. Following the inception in January 2006 of the revised advanced level mathematics curriculum, the use of GCs is incorporated in assessment for all mathematics subjects known as H1 Level Mathematics, H2 Level Mathematics and H3 Level Mathematics respectively, H1 level being the lowest, and H3 level the highest. In other words, all pre-university students are now expected to use a GC during public examinations and accordingly all main stream schools offering mathematics subjects at the advanced level, namely junior colleges, are now expected to fully integrate the GC into the mathematics classroom.

The impact of the decision to integrate the use of GCs in the advanced level mathematics curriculum (hereafter the new curriculum) has been considerable: for schools, it means restructuring curricular programmes and refining modes of assessment so that they incorporate the use of GCs; for teachers, it means acquiring new skills in utilising the GC as well as improving classroom pedagogy to include instruction on GC use and to harness the power of GCs in teaching; and for students, it means acquiring the skills in using a handheld technology more advanced than the scientific calculator, with which they are so familiar, in addition to mastering new concepts and skills in mathematics and integrating the two sets of skills so as to enhance problem solving especially during assessment.

The changes that school leaders, curriculum planners, teachers and students need to manage for successful integration of GCs into the curriculum clearly bring a number of challenges along with them. Against this backdrop, this paper shall begin with a delineation of the rationale for the integration of the GC into the new curriculum. A brief discussion on the impact of GC integration on the Further Mathematics curriculum will follow, since the move to allow the use of GC in Further Mathematics is a stepping-stone to making GC use an integral part of the new curriculum. As teachers play a crucial and pivotal role in the successful implementation of a GC-infused curriculum to bring benefits to students in the mathematics classroom, we shall also discuss the professional development and training of teachers in the acquisition of GC skills and the pedagogy of using GC in teaching and learning mathematics. Lastly, the impact of the GC on assessment in the new curriculum will be discussed.

Rationale for Integrating the GC into the Curriculum

One of the main emphases of the advanced level mathematics curriculum has been the acquisition and application of mathematical concepts and skills. While the new curriculum continues to emphasise this, there is now an even greater focus on the development of students' abilities to conjecture, discover, reason and communicate mathematics with the aid of technological tools, in particular the GC. More precisely, through integrating the GC into the new curriculum, it is hoped that the following objectives can be achieved:

(1) Allow students access to a wider range of problems.

Though non-CAS GCs cannot perform symbolic manipulations and computations, they are capable of performing a wide range of complex numerical computations such as numeric integration, solving of equations numerically and matrix operations. With each student equipped with a portable and powerful device such as the GC, a wide range of problems can now be discussed in the classroom and be included in assessment. Indeed, students can now deal with problems such as those which involve computing definite integrals of functions whose integral is not expressible in closed form or those which require solving of higher order polynomial equations. This will in turn open up greater teaching opportunities and expand students' learning horizon through more realistic applications and activities.

(2) Allow students more time on mathematical investigation, thinking, reflection and making inferences through exploratory work and experiments

In the new curriculum students will learn to work collaboratively with others in exploratory work and experiments with the aid of the GC, share ideas and discuss their findings. They will also learn to pose problems, to communicate the solutions mathematically, and to discuss alternative solutions. Using the GC in the classroom thus creates opportunities for students to engage in active learning. It also allows student to learn mathematics in more practical and meaningful contexts. As the GC enables students to execute routine computations and procedures quickly and accurately, students can spend more time on investigating mathematical concepts, observing patterns and making inferences. For instance, in solving problems such as the one below, students can, with the aid of a GC, examine the behaviour of the given function by graphing it for different values of k with relative ease, thus allowing them more time to think and reflect on their discovery, make conjectures and verify their conjectures (Ng, 2006d).

Example 1

A function *f* is given by $f(x) = \dfrac{kx+1}{x+2}$ where *k* is a real number. Find the set of values of *k* for which the function *f* is monotonically decreasing in the interval $(1, \infty)$ and is such that *f*(*x*) is not negative for some $x \in (1, \infty)$.

(3) Allow students to make connections between algebraic and geometric ideas

With the aid of the GC, mathematical concepts such as differentiation and integration which are often dealt with analytically can now be introduced in a more visual manner as the GC can be used to provide visualisation of the concepts in the form of graphs. Indeed, the behaviour of a function can be better understood as students, equipped with a GC, move between numerical, graphical and algebraic representations of the function by simply switching between screens. It is noteworthy that in a study by Ruthven (1990) in which the results of students that had access to a GC and those who did not through a two-year course were compared, it was found that those who used the GC regularly performed significantly better in linking the algebraic and graphical representations of a given function.

Wheatley (1997) argues that the distinction between good and poor problem solvers is the extent to which they utilise visual methods in the problem solving process. Indeed, the ability to utilise the graphing functions of the calculator to visualise problems and to make connections between different representations of a mathematical concept enhances one's problem-solving skills. To elucidate this point, we consider the example below:

Example 2

Prove that $\dfrac{a^4+b^4}{2} \geq \left(\dfrac{a+b}{2}\right)^4$ for all $a > 0$ and $b > 0$.

As illustrated by Ng (2006d), without the aid of any technological tools, students would be inclined to prove the above inequality algebraically, perhaps by factorising $\dfrac{a^4+b^4}{2} - \left(\dfrac{a+b}{2}\right)^4$ into a product of non-negative factors, namely $\dfrac{1}{16}(7a^2+10ab+7b^2)(a-b)^2$. With access to a GC, students could adopt a more graphical and geometrical approach to solving this problem by considering the behaviour of the function given by $y = x^4$. The desired inequality follows readily from the convexity of the function $y = x^4$. Such an approach also allows students to generalise the result by considering a similar property for other convex functions.

(5) Enhance students' problem solving skills

Problem solving is the crux of the Mathematics curriculum in Singapore and it is therefore integral that students learn to utilise the GC at each stage of the problem solving process in order to fully harness its capabilities. Having access to a GC allows students to examine various cases of a problem situation in a way that is both speedy and precise. It also provides a means for students to identify patterns and relationships between variables, information from which they may generate possible solution methods and strategies to solve the problem. Precious time that has been formerly devoted to tedious paper-and-pencil computations can now be redirected to the development of problem solving strategies and thinking skills. The GC also enables students to check and edit errors with considerable ease.

A number of researchers have explored the role of the GC in problem solving, the benefits it affords and the difficulties that may arise in its usage (e.g. Kendal and Stacey, 1999; Hong, Toham and Kiernan, 2000). More recently, Katsberg and Leatham (2005) investigated the success of GC usage in problem solving and have found that it depends on a number of factors. At the basic level, students need to have access to GCs and learn concepts through classroom pedagogy that successfully integrates GC use. The extent to which the technology is integrated into curriculum also plays an important role in the success of problem solving using GCs. In addition, students' response and approach to problem solving using the GC is dependent on the pedagogy with which they are taught.

The importance of pedagogy in integrating the GC into the classroom was also demonstrated in a study by Kendal and Stacey (1999) where the pedagogical approaches of three teachers in the instruction of calculus to Year 11 students were examined. It was found that students of the teacher who utilised an interactive problem solving approach and placed greater emphasis on the links between different representations with the aid of the GC demonstrated improved discernment when using the GC during assessment. There was also an observable reduction in conceptual error when compared to the students who were taught by the other two teachers, one of which emphasised calculator and algebraic approaches in a parallel manner, while the other focused on the utilisation of the GC for its graphing functions.

In a nutshell, merely instructing students in the use of a GC does not provide sufficient experience that allows them to develop effective problem solving skills using the GC. Teachers therefore have the responsibility to acquire the necessary GC skills and develop the pedagogy of using the GC in teaching and learning that allows them to impart these skills to their students. In a later section we will discuss the training of teachers in the effective integration of GC into mathematics teaching.

Impact of Integration of the GC into the Further Mathematics Curriculum

As aforementioned, the use of GCs in public examinations was first permitted in 2002 for Further Mathematics, a subject taken by students with a keen interest in, or who have an aptitude for, mathematics. Though GCs were allowed for use in the public examination it was made clear to the junior colleges that the use of GCs in examinations was not expected in the sense that examination questions would be "GC-neutral," i.e. the questions would be designed in such as a way that candidates who did not have access to a GC would not be disadvantaged. This move was in a way an interim measure, in the Education Ministry's plan to implement a GC-infused curriculum, so as to allay anxiety of the students and teachers in having to master the use of a GC and at the same time provide the junior colleges some lead time in getting ready for the comprehensive implementation of a GC integrated curriculum in 2006. As a result, though the top rung junior colleges did attempt to prepare their students in the use of GCs so as to gain some advantages during the Further Mathematics examinations, most of the other junior colleges chose the status quo.

Inevitably, the introduction of GC in Further Mathematics has resulted in some changes in the design of the Further Mathematics examination questions. While no significant changes have been observed in questions on topics such as Mathematical Induction, Summation of Series, Roots and Coefficients of Polynomial Equations, Differentiation, Integration, Differential Equations, Reduction Formula, Complex Numbers, and Vectors, marked changes are seen in questions on Curve Sketching as well as Polar Coordinates. Generally, questions are now designed to test students more on their understanding of concepts than numerical or symbolic computations and manipulations and fewer marks are awarded on sketching of curves. For questions on Linear Spaces and Statistics, fewer questions involving computations which can be done on a GC have been set. In other words, attempts have been made to make the questions GC-neutral.

To illustrate the changes, we shall compare some typical questions before and after the introduction of the GC in the Further Mathematics curriculum. For example, before the introduction of the GC, a typical question on curve sketching looks like the one below in which parts (ii) and (iii) are clearly not GC-neutral.

Example 2

The curve C has equation $y = \dfrac{x^2 - 2x - 3}{x + 2}$.

i) Find the equations of the asymptotes of C.
ii) Draw a sketch of C showing the asymptotes and the coordinates of the points of intersection of C with the axes.
iii) On the same diagram draw a sketch of $y = \dfrac{4}{(x+4)^2}$.
iv) Hence show that the equation $x^4 + 6x^3 - 3x^2 - 60x - 56 = 0$ has exactly 2 real roots.

After the introduction of GC, examination questions such as the following in which the equation of the curve is defined in terms of some unknown parameters rather than numerical values have been set. Note that even though this question is supposed to be GC-neutral, students who have access to a GC will still have an advantage in answering part (iv) as they can still graph C using a GC by putting say a = 2, b = 3 and c = 4.

Example 3

The curve C has equation $y = \dfrac{(x-a)(x-b)}{x-c}$, $0 < a < b < c$.

i) Express y in the form $x + P + \dfrac{Q}{x-c}$, giving the constant P and Q in term of a, b and c.
ii) Find the equations of the asymptotes.
iii) Show that C has two stationary points.
iv) Given that $a + b > c$, sketch C showing the asymptotes and the coordinates of the points of intersection of C with the axes.

On the other hand, for the topic on Polar Coordinates, a typical question before the year 2002 is close to the question below which is clearly not GC-neutral, or GC-biased which it is also called, as the GC is capable of performing numerical integration.

> **Example 4**
> Find the area of a loop of the curve whose polar equation is $r = a\sin 4\theta$, where a is a positive constant.

Following the introduction of GC, students have to answer questions with more parts which require them to show or prove that certain properties or results hold true, a consequence of the attempt to make the questions more GC-neutral. One such question is as shown below.

> **Example 5**
> The curve C has polar equation $r\theta = 1$, for $0 \le \theta \le 2\pi$.
> i) Use the fact that $\frac{\sin\theta}{\theta}$ tends to 1 as θ tends to 0 to show that the line with Cartesian equation $y = 1$ is an asymptote to C.
> ii) Sketch C.
> The points P and Q on C correspond to $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{3}$ respectively.
> iii) Find the area of the sector OPQ, where O is the origin.
> iv) Show that the length of the arc PQ is $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{1+\theta^2}}{\theta^2}\,d\theta$.

To investigate the usage of GC among Further Mathematics students, a study was carried out in 2003 with 190 second year pre-university students who take Further Mathematics at a junior college (Ng, 2006c). The study comprised three surveys carried out on three occasions of assessment: the ‘common test’ in June which was actually a mid-year summative assessment, a lecture test in August, and the preliminary examination in late September which was the final formal assessment for the two-year junior college programme. The topics involved in the three surveys were Curve Sketching, Linear Spaces and Statistics respectively. The first survey was carried out when students attempted a question on curve sketching that required them to draw a sketch of the curve whose equation was given, labelling the asymptotes, stationary points and intersections with coordinate axes (see Table 1 below). After answering the question, students were requested to indicate whether they utilised a GC. From the survey, students who used a GC appeared to perform better, with 61% of them scoring 4 or 5 marks out of a possible total of 5 marks while only 42% of the students who did not use a GC scored 4 or 5 marks. On the other hand, only 3% of the students who used a GC scored 0 or 1 mark while the corresponding percentage for those who did not use a GC is 24%.

In the second survey, students were asked to indicate whether they spent a significant amount of time on answering a Linear Spaces question, which entailed relatively intensive computations. While only 5% of the students who utilised the GC in answering the question stated that they spent a lot of time on the question, among those who did not use the GC, 31% indicated that they spent a lot of time on the problem. While this provides some indication of the temporal benefits of using the

GC in attempting advanced level questions under timed conditions, the subjective nature of "a lot of time" does not allow for definitive conclusions. The third and final survey examined the performance of students in the second paper of their preliminary examination which was on Statistics. The results show that students who utilised a GC tended to perform better, with 13% of them scoring more than 70 marks out of a possible total of 100. In contrast, only 9% of those did not use a GC scored more than 70 marks.

Table 1: The surveys and their related test items.

<table>
<tr><th>Survey</th><th>Question</th></tr>
<tr><td>1</td><td>
<u>Curve Sketching (June Common Test Question 6)</u>
(a) The curve C has equation given by $y = x + \beta + \frac{\beta}{x}$, where $\beta \neq 0, x \neq 0$.
(i) Find the set of values of β for which the curve C cuts the x-axis at two distinct points. [2]
(ii) If $1 < \beta < 2$, draw a sketch of C, labelling clearly the asymptotes, stationary points and any intersections with the coordinate axes, if applicable. [5]

[Please indicate on the cover page whether you made use of the graphing calculator.]
</td></tr>
<tr><td>2</td><td>
<u>Linear Spaces (Lecture Test)</u>

Given that x is an eigenvector of each of the square matrices A and B with the corresponding eigenvalues being λ and μ respectively. Show that x is an eigenvector of
(i) kA,
(ii) A + B,
and find their corresponding eigenvalues. [4]
Find the eigenvalues and the corresponding eigenvectors of the matrix A, where
$$\mathbf{A} = \begin{pmatrix} 1 & -3 & -3 \\ -8 & 6 & -3 \\ 8 & -2 & 7 \end{pmatrix}.$$ [6]
Hence, find a matrix P and a diagonal matrix D in terms of n such that SP = PD where
$\mathbf{S} = \mathbf{A} + 2\mathbf{A} + 3\mathbf{A} + \ldots + n\mathbf{A}, \quad n \in Z^+$. [5]
</td></tr>
<tr><td>3</td><td>
<u>Preliminary Examination Paper 2 (Question 7)</u>

A certain local authority was looking into the length-of-service characteristics of its employees. Jobs were classified as 'manual', 'technical' or 'administrative'. Records were available showing how long each employee had served with the authority.
A total of 150 employees, chosen at random from all the employees of the authority, were investigated and the results were as follows:
<table>
<tr><th>Type of Jobs</th><th colspan="3">Length of service</th></tr>
<tr><td></td><td>< 6 months</td><td>6 months to 2 years</td><td>> 2 years</td></tr>
<tr><td>Manual</td><td>30</td><td>11</td><td>19</td></tr>
</table>
</td></tr>
</table>

Technical	13	13	20
Administrative	9	8	27

Examine whether the data provide evidence of an association between the types of job and the length of service of the employee at the 1% significance level. [6]

The above results suggest that GC users may perform better than non-GC users in a timed paper-and-pencil assessment. However, there were also many students who did not do well despite having the advantage of using GC technology. This might stem from the relative unfamiliarity with some GC functions in addition to the lack of exposure to specific methods of using GC in solving mathematical problems among these students. Further in-depth studies need to be performed to identify the factors that influence GC usage, such as teacher proficiency in GC and pedagogical practices. A teaching scheme that incorporates the use of the GC on a regular basis needs to be developed and systematically carried out to unfold the many facets of GC use and its potential as a teaching and learning tool.

Support and Training for Teachers

Since the implementation of the new curriculum, there has been considerable expectation among policy makers and educators alike of the success of GC use and the new possibilities that it affords both students and teachers, particularly in the classroom environment. However, informal feedback from the teachers suggests that GC is currently underutilised among students. This is consistent with research carried out in North America, Europe and Australia which has shown that students tend to utilise the GC like a scientific calculator, albeit one with more advanced computational functions (Graham, Headlam, Honey, Sharp and Smith, 2003). There is therefore a need to promote the use of the GC among students and teachers play a pivotal role. Indeed, whether teachers could effectively integrate the use of GC into the mathematics classroom so that their students can harness the GC in problem solving is central to the success of integrating GC into classroom.

In order that teachers could master the skills in using the GC and pedagogical repertoire to take advantage of the GC's capabilities, the issue of professional development needs to be addressed. Teachers need to have access to programmes that enable them to upgrade their skills. In-service courses on the use of GCs in the classroom conducted by the author in 2005 and 2006 have served the purpose of equipping teachers with the skills in using the GC as well as the pedagogy to use GC in the classroom. Participants of the courses were exposed to different pedagogical approaches with the GC to assist them in conducting lessons that use an inquiry-based approach and which are more student-centred (Ng, 2005). Feedback regarding the course suggested that teachers enjoyed the practical and hands-on approach to mastering the use of a GC and the ample opportunities given to apply the GC skills in solving advanced level problems. They were also more confident in using the GC and less apprehensive about integrating GC use into their teaching after attending the course. In a nutshell, teachers need to upgrade their professional skills through programmes that focus on the functions of the GC, its use in solving advanced level problems using a hands-on approach, which models the approach they should take with their students. As local teachers deem availability of teaching resources important to the success of their teaching, they also appreciate the provision of materials such as the book written by the author (Ng, 2006) that could be used both as a quick reference in their teaching or be used as a course book for getting their students started with the GC.

Teachers' attitudes towards the GC depend on how much they believe in the benefits that the technology can bring forth. They need to be made aware that such technology has the potential to promote a dynamic classroom environment and increase students' levels of confidence through better understanding of concepts and an increase in problem solving abilities (Dunham, 1993). In a study done by the author the factors that might influence the integration of Information and Communications Technology in the classroom were examined with the aid of a survey developed to garner responses of teachers regarding the matter (Ng, 2006a). The pilot study, based on the responses of 60 pre-service mathematics teachers, showed that for this group of teachers, usefulness and worthiness of the technology being integrated was the most important, followed by support from the various departments in the school and then availability and accessibility of the technology. In addition, professional development was also found to be an important factor. Although the response of this group of teachers may not be representative of the views of pre-university level mathematics teachers, it nonetheless shed some light on issues that are of utmost concern to them in the integration of technology such as the GC into their lessons. Teachers therefore need to be convinced of the merits of using the GC, even though the requirement for students to learn to use the technology is compulsory. Support from school administrative personnel and members of other departments are also vital as role models are required to assist teachers in considering the benefits of technology integration (Cafolla and Knee, 1999).

Impact of GC Integration on Assessment in the New Curriculum

As aforementioned, in the new curriculum the use of GC during advanced level examinations is not only allowed but is in fact expected as there will be questions which specifically require the use of a GC to answer. It will therefore be assumed that candidates have access to a GC during examinations. As a general rule unsupported answers obtained from a GC are allowed unless the question states otherwise. Where unsupported answers from a GC are not allowed, students are required to present the mathematical steps using mathematical notations and not calculator commands (SEAB, 2006). It is therefore imperative that students be fully aware of the circumstances under which they are expected to operate the GC during the examination. To help students acquaint themselves with the use of such technology under examination conditions, assessment needs to be modified at the school level in order to address this new development and teachers need to ensure that ongoing assessment, whether formal or informal, makes use of the capabilities of the GC as experience is a valuable commodity in the use of GC.

Generally, examination questions will be designed in ways to integrate GC in an appropriate and meaningful way so that candidates are free from carrying out tedious computations, thus creating more time and space for higher order thinking. Students may be asked to solve problems arising from a context or application where an analytical or neat solution is not available and where a numerical answer obtainable with the aid of a GC is necessary to take the question further or reach a meaningful conclusion. Students may also have to carry out simple (but otherwise tedious) investigations or trials so that they may form a hypothesis which they could be asked to prove analytically.

The design of a question is determined by its assessment objectives. If the objective of the question is to assess if students have understood a particular concept, then the question may be designed in such a way that the answer is not obtainable from the GC directly, such as when the magnitude of the value is too large or small for the GC to handle. Using a parameter instead of a numerical value is another possible approach. In some questions, students will be expected to be able to reason why certain mathematical properties hold.

One of the concerns of teachers pertains to the use of the GC in curve sketching. Based on the objective of the question, that is, to test skills in determining the properties of a given curve as opposed to deriving an equation of a graph, teachers need to formulate their questions accordingly. The emphasis on actual sketching of curve has been reduced and the focus is on the properties of the curve. However, students are still expected to be familiar with standard forms of certain graphs. Thus, questions can still require students to perform computations to solve a problem. For example, a standard curve is given and students are required to derive the equation of the graph. When teachers wish to request that students use an analytical method in a question, they may indicate this using a number of phrases, such as “use a non-calculator method”, to ensure that students arrive at a solution via the analytical method.

There is also heightened concern that the GC is used as a medium for trial and error rather than mathematical reasoning. While it is inevitable that some questions require some form of trial and error, particularly as a stepping stone to solving the problem fully, students need to be made aware that GC use does not replace the requirement for conceptual understanding and mathematical reasoning. The use of the GC as an initial investigative method via trial and error is a valid one which can lead to further explorations and discovery through reasoning. So while questions could be designed to allow students to explore various cases, look for a pattern, and make conjectures, it should be designed in such a way it cannot be answered using entirely trial and error method and that more feasible method which requires a demonstration of mathematical skills are available.

Another area of concern arises from the requirement to clear calculator memory prior to the start of the examination in order to erase any programmes or applications which are either pre-loaded or downloaded by students for learning purposes. This is a practical concern as the scale of national examinations is fairly large and not all the invigilators are mathematics teachers who are familiar with the GC. This issue has been partially addressed by ensuring that all invigilators are provided with the instructions on resetting calculators.

Conclusion

Advocates of GC use generally reason that using the GC in the classroom will reduce time spent on by-hand calculations and as a result, classroom time may be redirected to investigation and understanding of mathematical concepts (Drijvers and Doorman, 1996) and research has shown students can benefit from the use of GC in terms of academic achievement (e.g. Harskamp, Suhre and Van Struen, 2000; Hong, Toham and Kiernan, 2000). Indeed, integration of the GC into the mathematics curriculum allows students and teachers the opportunity to explore mathematical concepts through novel approaches that might otherwise be unavailable through other means. A wider range of problems are now accessible to students, such as non-standard functions and matrices, which create greater opportunities for teachers to further extend student’s learning through meaningful classroom activities. The intent of GC use is to encourage students to engage in stimulating discussions and activities in mathematics where they can explore different approaches in solving problems and establish links between different concepts. It is an integrated approach in learning of mathematics as opposed to a skill that is appended to supplement learning on the side. Furthermore, as reasoned by advocates of GC, the GC allows students to execute routine procedures accurately and quickly, allowing for the generation of connections between algebraic and geometric ideas and rapid switching between different representations when exploring mathematical results. This allows more time for students to think and reflect as well as discover different concepts. Such benefits do not only apply to activities in the classroom but in assessment as well.

The GC should be harnessed so that students can learn mathematics in a meaningful context, using analytical, graphical and numerical techniques which allow them to appreciate the practical nature of their endeavours and appreciate the application of mathematics to real-world problems and situations. These capabilities of the GC will enable students to learn through exploration and help them investigate mathematical concepts with greater ease. Students will also be encouraged to communicate with others and learn through collaborative work, share and discuss their findings on solutions to real-life problems and understand mathematical relationships through in-class activities.

The potential for student conceptual understanding, skill acquisition and mathematical reasoning development that comes through usage of the GC is only limited by the extent to which this technology is utilised in the classroom. Teachers need to provide sufficient instruction, scaffold students' learning processes and create more avenues of exposure so that students are able to learn how to use the GC throughout the various stages of the problem solving process rather than use it to replace complex manual computations. However, rigorous mathematical solutions cannot be replaced by GC use. Conceptual understanding, mathematical reasoning and skill proficiencies are key elements of the curriculum which should not be downplayed by the use of GC.

The modes of assessment, particularly in the setting of questions, need to be modified to reflect the changes in the curriculum and national assessment requirements. Teachers therefore have to be certain about the objective of their assessment and modify their requirements accordingly. Furthermore, it is important to note that as with all pedagogical approaches, with technology or otherwise, there are limitations pertaining to its use. Therefore a holistic approach to using the GC should also include creating awareness and identifying potential limitations of GC use.

Conclusively, integrating the GC into the advanced level curriculum proves to be a challenge at the various implementation levels, but it is nonetheless a worthy undertaking, especially in the light of the benefits that the GCs can bring forth, particularly on mathematical achievement (Ruthven, 1995). However, the use of GC should not be treated as a means to an end, be it a gain in academic results or skills in technology, but rather as a tool that requires circumspect usage in order to harness it to its full advantage.

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