Development of the Active Teaching Materials in Mathematics Education

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Abstract: When considering a human model, the model must be viewed as a process model, as a knowledge model, and as a control model. We can consider that such an approach has been applied to the understanding of theorem proving. Humans appear to use two types of knowledge to understand, investigate and act. The human model for understanding in mathematical thinking is based on a three-level model of human action. In cognitive science, human beings use strategies to solve problems. Strategies are also used when human beings solve mathematical problems. We used a three-level human behavior model to analyze the targets that appear during problem solving and the strategies used to solve the problems Development of the teaching materials to help activity of students in university mathematics education has a lot of attention. The computer technology promoted the use of efficient software within mathematics education. In this paper, we show a practical example of geometry theorem proving as a training example of knowledge base and consider the effectiveness of using computer in university mathematics education.

1. Introduction

Human modeling can elucidate how humans make decisions and how learning ability can be improved. Techniques that have been used widely in this field can be classified into those that integrate a control theory model ([16]), a probability theory model ([1]), a will determinism model ([6]), a processing model ([2]), or an artificial intelligence model ([9]), and those integrating all of these models ([15]). The rapid development in research on computer technology, cognitive psychology, and artificial intelligence has led to a shift from models ([14]), which ignore the psychological processes of behavior, to information processing models ([8]) and artificial intelligence models.

However, these models are still limited with respect to human cognition and while promising new research is being conducted, such research ([4]) has not been collated into a theoretical system with specific methods. When considering a human model, the model must be viewed as a process model, a knowledge model, and a control model. At present, the models used to describe human information processing include a 7-step user action model, a 3-level human action model, a human cognition model, and a knowledge model. Unfortunately, there is currently no optimal model of understanding in cognitive science. When constructing a new model of human cognition, the effects of various factors should be accounted for. However, attempting to construct a model based on the consideration of as many factors as possible would complicate the model with too many hypotheses and parameters, making it difficult to test.

Therefore, the human model proposed in the present work is based on the three-level model of Rasmussen ([12]) which arose from research on human actions in large-scale systems such as an atomic energy plant or an aircraft. We can consider that such a model has been applied to understanding in cognitive science. Thus, this model was used to position problem-solving strategy and clarify cognitive changes. Almost all human cognitive activities arise from an interactive process between information from the outside world and the knowledge possessed by the individual. Information “from the outside world” is affected by the cultural background and social customs. “Knowledge possessed by the individual” is influenced by the experiences of the individual.

Despite this complicated and continuous interactive reaction between external and internal factors, there are environments and stimuli common to all humans; therefore, common human reactions exist. Internal knowledge includes not only linguistic and analytical intellectual knowledge, but also non-linguistic and comprehensive intellectual knowledge. Humans seem to use both types of knowledge to understand, learn, and act. Determining whether such human activities can be accurately evaluated and included in a theoretical framework is the key to developing a user interface that can elicit the full intellectual potential of humans.

If we want to conduct effective cognitive science that identifies the source of understanding and creativity, rather than simply utilize unconscious human actions to reach into the depths of human psychological activities, then we must establish a framework that explains the communication between the external and internal sources of knowledge. Our proposed human model of understanding in cognitive science is based on a three-level model of human action. Human researching begins with physical processes, followed by visual processes, and finally intuitive (symbolic) processes. The symbol-oriented problem-solving method requires focusing on a problem for a long time with carefully planned procedures. Therefore, people who think in this way are often successful in reaching a conclusion.

In rule-based actions, classifying behavior occurs in response to problems. This makes efficient search possible, by indicating what should be done next based on the present situation. To examine how the classification is expressed and constructed, subjects are given several problems to solve for the present study. If the calculation procedure is incorrect when the behavior classification is being constructed, the results will not be predictable. What required identification was how easy it was to reconstruct the classification and what conditions facilitate the identification of the actions that were the cause of the error. In order to assess the degree of freedom achieved, a measurement method was required for identifying the necessary conditions for moving from a rule-based activity to a skill-based activity. Knowledge-based behavior results from cognition and interpretation of external conditions and the construction of a psychological model that uses skill- and rule-based behavior as a solution process. This requires knowledge of how humans solve problems. Information must be organized and recorded for cognition and understanding. Receiving a message means that the message is reconstructed. In order to do this, one must have the resources with which to conduct this reconstruction. An agent should be able to function appropriately with human common sense and the ability to learn. The agent should always consider safety and accuracy and also possess the ability to explain what is dangerous. The issues in designing such an agent and its actual use must be identified. Mistakes in knowledge-based behavior due to illusions or uncertain memory can lead to the inability to choose or the lack of knowledge of an operation procedure. To integrate these skill-, rule-, and knowledge-based behaviors, the three-level human action model was painstakingly reconstructed in order to establish a useful human model, which is used for the learner in cognitive science.

Gaining knowledge and solving problems is two side of the same coin. Reasoning is a reduction of the complex to the simple. Mathematics is the science of "explanation" (reducing everything to "plain things"). Using computer algebra system, we can reduce the geometry theorem proving. The development of their material is an exciting study of mathematics education.

2. Human Model

According to the three-level human behavior model of Rasmussen, automatic human actions can be classified into the three levels of skill-, rule- and knowledge-based actions (Fig. 2.1, [13]).

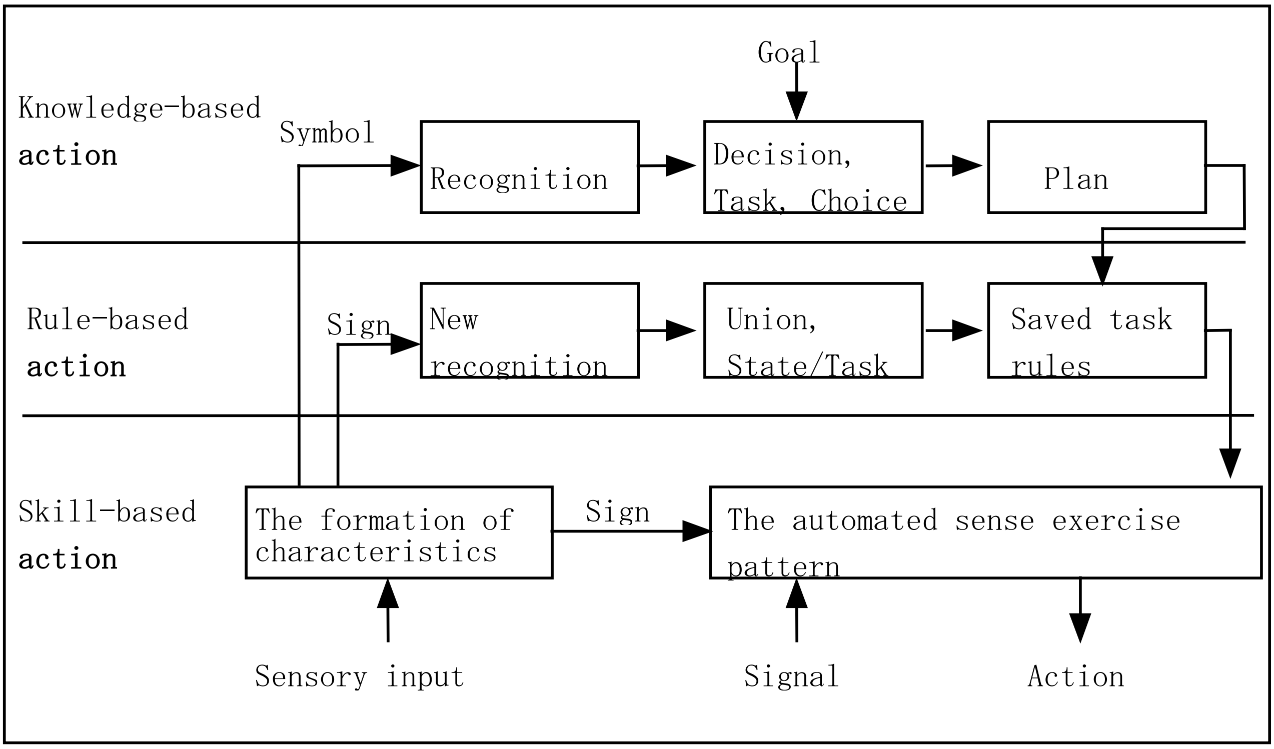

Figure 2.1 Three-Level Model of Human Action

Skill-based action: These actions occur at the stage when some intention is formed. However, the actions are automated and are executed without the control of an action pattern.

Rule-based action: Actions that utilize a previously acquired rule in order to realize a specified purpose. The learner constructs and executes a series of actions.

Knowledge-based action: The learner recognizes the situation, manipulates a psychological model, and considers possible strategies.

A skill-based action is a response that occurs in less than 1 second ([10]). A chain of skill-based actions is a rule-based action. Thinking about how to solve a problem is a knowledge-based action.

Skill-based actions are performed smoothly without intentional control. Rule-based actions require a great deal of repetitive practice in order to be transferred to the skill-based level. First, the external conditions must be recognized, then the rules for composing the act are combined with the conditions required to carry out the behavior. Knowledge-based actions require the recognition of external conditions, the interpretation of these conditions, the construction of a psychological model for considering solutions, planning, and finally, the use of the other two behavior levels to carry out the action. This is a process model in which mastery of behavior requiring thought is internalized to the point where it can be carried out unconsciously. Mistakes can be explained as omitted steps, or for example, as pushing the wrong nearby button in smoothly carried out skill-based actions. In the case of knowledge-based actions, illusion can lead to error. In the present study, this process was analyzed using Rasmussen's three-level human behavior model in order to identify what functions are essential to facilitating smooth action and learning. Behavior used to learn about problems and how to solve them is classified in detail according to the three-level model. Humans act by classifying issues and their relationships by consciously combining them. Humans control themselves by constantly observing, thinking about, evaluating, and integrating their behavior in order to achieve accuracy, continuity, consistency, and normality ([7]). Classified factors can be separated into the same three levels as the general actions.

3. Strategy

In cognitive science, humans use strategies to solve problems. Strategies are used as knowledge to plan solutions and decide procedures. When these procedures, in general or for the most part, obtain the correct answer, the procedure is called a heuristic; however, such heuristics do not always result in a correct solution. The study of factors related to the early stages of strategy is based on observing the results of an individual's problem solving method. Following the initial stage, the individual passes through a series of stages and then reaches a solution. However, consensus has not been reached regarding the meaning of these stages.

For a computer to solve a problem, the expression of the problem solution strategy must be accurately described as follows:

1. The problem must be expressed as a problem space.
2. The problem space consists of the possible operations available to change the premise conditions of the problem, the final conditions of the problem, and the middle conditions.
3. The problem space has a passage that can be used (dead end).
4. Solutions to the problem are to be sought from the viewpoint of problem-solving goals. They are not carried out from a trial and error perspective.

Strategies are used even when human beings solve mathematical problems. Recognition knowledge and experience are used as "doing it like this is effective in this case". The ability to rapidly reference knowledge is required for strategies based on experience. Furthermore, the recognition of thoughts and feelings controls. The famous book by the mathematician Polya, "How to solve it([11]), showed the processes of mathematical problem solving; however, one can not learn how to use heuristics in problem solving just by reading a book. In researching problem solving, there are two contrasting concepts. The first emphasizes insight, flash, and senses, while the second emphasizes experiential knowledge. The former concept employs a strong tendency to perceive that strategies of thought are learned through the experience of problem solving. In other words, it is assumed that an intuitive feelings and specific technical abilities can be acquired. In the latter concept, it is assumed that problem solving ability arises from the accumulation of rules inherent to the domain provided by an individual problem. Such differences depend on the problem's nature, domain, and level, and the type of person involved in the learning process. In addition, it is difficult to establish clear boundary lines between these two concepts. In problem solving, experiential knowledge plays a large role. Heuristics are general ideas or algorithms (a procedure providing the correct solution), and are widely used. Heuristics are equal to "the logic of a thought". Examples of extremely general strategies are "try to draw a figure if you come across a difficult problem", and "search for similar problems that you have experience with". There are also concrete strategies we are familiar with, such as "A problem requiring the comparison of quantities requires two differences, and a transform formula" and "try to make clauses that differ next to each other for number sum sequence problems" ([5]). Therefore, the kinds of strategies that students actually use were examined. The present research explores the differences in the learning of students who adopt the strategy of physical principles without knowledge of the usual learning methods for mathematics and students who adopt the natural strategy of mathematics, which proceeds with logical, progressive thought. These differences are assembled in the student's brain, which determines how these strategies are to be used. Finally, a learner independently searches for heuristics. It is desirable to understand this process, in order to recognize the meaning of cultivating natural mathematical comprehension and thinking power. However, currently, there are many cases in which the learner does not consciously study problems from the perspective that "there are various learning methods and heuristics".

4. Practical Use of Knowledge Base Evaluation in Teaching

Theorem proving needs various knowledge (skill base, rule base, knowledge base). In particular, geometry theorem proving has an image of figure and can give interest to learners. And recently, geometry theorem proving that used ideal theory on computer algebra system developed. Many students are thinking that there is not relation between geometry theorem proving and ideal theory. We can make clear how various knowledge is used by analyzing a new experience of students. In this section, we show a practical example of geometry theorem proving as a training example of knowledge base and consider the effectiveness of using computer in university mathematics education.

The basic idea underlying the methods we will consider is that once we introduce Cartesian coordinates in the Euclidean plane, the hypotheses and the conclusions of a large class of geometric theorems can be expressed as polynomial equations between the coordinates of collections of points specified in the statements. Here is a simple but representative example.

Example . ([3]) Let ABCD be a parallelogram, O be the intersection of the diagonals AC and BD. Show AO=OC. (see Figure 4.1)

The implicit assumption that the parallelogram is in a general position means that any three points among the four points A, B, C, and D can be arbitrarily chosen. Then we can let A=(0,0), B=$(u_1,0)$, C=(u_2,u_3), D=(x_2,x_1), and O=(x_4,x_3). The hypothesis equations are;

$h_1=u_1x_1-u_1u_3=0$	AB is parallel to DC
$h_2=u_3x_2-(u_2-u_1)x_1=0$	DA is parallel to CB
$h_3=x_1x_4-(x_2-u_1)x_3-u_1x_1=0$	O is on BD
$h_4=u_3x_4-u_2x_3=0$	O is on AC

The conclusion AO=OC is: $g=2u_2x_4+2u_3x_3-u_3^2-u_2^2=0$.

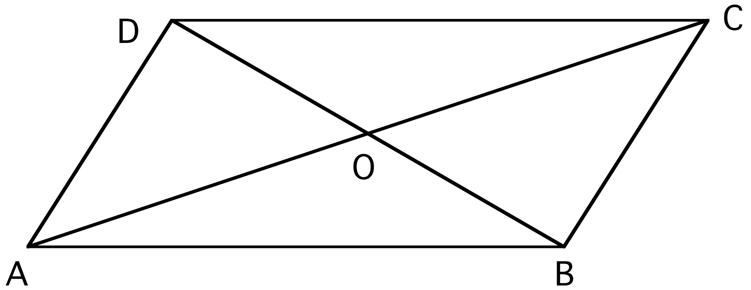

Figure 4.1

We use Groebner Basis method. Let $GB(h_1,h_2,h_2,h_4)$ be a Groebner Basis of h_1,h_2,h_2,h_4. Then $GB(h_1,h_2,h_2,h_4)=\{x_1-u_3,\ x_2-u_2+u_1,\ x_3-u_3/2,\ x_4-u_2/2\}$, which reduce the conclusion polynomial $g=2u_2x_4+2u_3x_3-u_3^2-u_2^2$ to zero. Therefore, I< h_1,h_2,h_2,h_4> include g, AO=OC is proved.

One of the important applications of Groebner basis method is to decide ideal membership for polynomial ideals. Paradoxically, students can feel result of learning of polynomial ideals.

A proof by using computer algebra system is as follows:.
(We use Maple)

> with(Groebner):
F := [u1*x1 – u1*u3, u3*x2 – (u2 – u1)*x1, x1*x4 – (x2 – u1)*x3 – u1*x1, u3*x4 – u2*x3];
F := [u1 x1 – u1 u3, u3 x2 – (u2 – u1) x1, x1 x4 – (x2 –u1) x3 – u1 x1, u3 x4 – u2 x3] (1)
> G:=Basis(F, plex(x1, x2, x3, x4, u1, u2, u3))
G:= [- u2 u1 u3 + 2 x4 u1 u3, - u3 x4 + u2 x3, 2 x3 u1 u3 – u1 $u3^2$, (2)
- u2 u1 u3 + u1 u3 x2 + $u1^2$ u3, 2 u1 x3 x2 – 2 x3 $u1^2$ + 2 $u1^2$ u3 – u2 u1 u3,
x1 u2 – u3 x2 – u1 u3, u1 x1 – u1 u3, x1 x4 – x3 x2 + x3 u1 – u1 u3]
> factor(G)
[- u1 u3 (- 2 x4 + u2), - u3 x4 + u2 x3, - u1 u3 (- 2 x3 + u3), u1 u3 (- u2 + x2 + u1), (3)
- u1 (2 x3 x2 – 2 x3 u1 + 2 u1 u3 – u2 u3), x1 u2 – u3 x2 – u1 u3, u1 (x1 – u3),
x1 x4 – x3 x2 + x3 u1 – u1 u3]
> with(Groebner):
F1 := [u2 - 2*x4, - u3*x4 + u2*x3, - 2*x3 + u3,   – u2 + x2 + u1,
2*x3*x2 – 2*x3*u1 + 2*u1*u3 – u2*u3, x1*u2 – u3*x2 – u1*u3, x1 – u3,
x1*x4 – x3*x2 + x3*u1 – u1*u3];
F1 := [- 2 x4 + u2, - u3 x4 +u2 x3, - 2 x3 + u3,   – u2 + x2 + u1, (4)
2 x3 x2 – 2 x3 u1 + 2 u1 u3 – u2 u3, x1 u2 – u3 x2 – u1 u3, x1 – u3,
x1 x4 – x3 x2 + x3 u1 – u1 u3]
> G1:=Basis(F1, plex(x1, x2, x3, x4, u1, u2, u3))
G1:= [- u2 + 2 x4, 2 x3 – u3, - u2 + x2 + u1, x1 – u3] (5)
> g := 2u2*x4 + 2u3*x3 – u3^2 – u2^2

$$g := 2\ u2\ x4 + 2\ u3\ x3 - u3^2 - u2^2 \qquad (6)$$

> $x3 := \frac{u3}{2};$

$$x3 := \frac{1}{2}\,u3 \qquad (7)$$

> $x4 := \frac{u2}{2};$

$$x4 := \frac{1}{2}\,u2 \qquad (8)$$

> g;

$$0 \qquad (9)$$

The students who solved above (1)~(2) are understanding fundamental technics for membership problem of polynomial ideals. These students have knowledge of rule base on this elucidation in this question. The students who got discovering (3)~(5) achieved learning result of knowledge base. (u1 is not equal to 0 and u3 is not equal to 0) is a condition for holding a parallelogram. An ability to process this condition is understand by knowledge base. We were able to get these results by interviewing to students. For analogous questions, the answer of students showed similar results. Understanding of students shows various aspects. However, it is effective as a method to evaluate understanding of students who learned the theory of polynomial ideals.

5. Conclusion

In the three-level model of human behavior, operations and strategies can be identified and considered in relation to human thought processes in order to facilitate error-free problem solving. In consideration of surface features and conditions, similar problems can be recognized and suitable problem-solving methods can be identified. In addition, it was found that contents of the subconscious can be raised to the knowledge-based action level in order to support the expression process and the achievement of efficient functioning.

Strategic use requires the student to think about the problem and the tool. To reach this stage, students must encourage to develop the habits of discerning and strategic use of computer algebra system along with the elements of algebraic insight required. In this way, computer algebra system gives many opportunities for rich mathematical learning. Technology in mathematics education is the fulfilment of the aspiration of mathematics education in the computer age. By recent advances in computational logic, computational algebra and software technology, the automation of reasoning promises to become practical feasible and useful for mathematics education.

References

[1] Brunswik, E.: Representative design and probabilistic theory in a functional psychology. Psychological Review 62 (1955).

[2] Card S., Moran T. P., and Newell A.: The Psychology of Human-Computer Interaction. Lawrence Erlbaum Associates (1983).

[3] Chou, S.-C.: Mechanical Geometry Theorem Proving, D. Reidel Publishing Company, Dordrecht (1991).

[4] Clancey W.: The frame of reference problem in the design of intelligent machines. Architectures for intelligence: The 22nd Carnegie symposium on cognition. Lawrence Erlbaum Associates (1991).

[5] Ichikawa S.: Psychology of Learning and Education, Iwanami Shoten, 91 (1995) (in Japanese).

[6] Kahneman, D. and Tversky, D.: Prospect theory: An analysis of decision under risk. Econometrica, 47(2) (1979).

[7] Kozuya T. (ed.): Memory and Knowledge (Cognitive Psychology Lecture 2), University of Tokyo Press, 17 (1978) (in Japanese).

[8] Neisser U.: Congnitive psychology. Prentice-Hall (1967).

[9] Newell A.: The knowledge level. Artificial Intelligence, 18 (1982).

[10] Polson P. G. and Kieras D. E.: A Quantitative Model of the Learning and Performance of Text Editing Knowledge, Proceedings of ACM CHI'85 Conference on Human Factors in Computing Systems. (1985).

[11] Polya G.: How to solve it. Doubleday (1957).

[12] Rasmussen J.: Information Processing and Human-Machine Interaction, Elservier Science Publishing (1986).

[13] Rasmussen J.: Recognition engineering of interface, Keigakushuppan (1990) (in Japanese).

[14] Skinner, B.F.: Science and human behaviour. FreePress New York (1953).

[15] Tulving E.: Organization of memory, Quo vadis? The cognitive neuro-science, MIT Press (1994).

[16] Wiener N.: Cybernetics. John-Wiley & Sons (1948).