

Modeling in Middle and High School Mathematics

Zhonghong Jiang

zj5@nyu.edu

Department of Teaching and Learning
New York University
U.S.A.

Abstract: This article is to illustrate the rationale and ideas of developing mathematical modeling oriented instructional materials to improve middle and high school students' mathematics learning. The materials will consist of two sets of interrelated modules—Instructional Modules for Middle and High School Students (M-S) and Professional Development Modules (M-PD) for their teachers. These modules will meet the following criteria: using rich problem-solving tasks to reinforce and expand students' understanding of root concepts in algebra and geometry, having the potential to integrate required curriculum topics in a more coherent manner, emphasizing real-world mathematics and the use of technology, and being appropriate for a diverse group of students and teachers.

1. Introduction

A critical challenge for U.S. mathematics education is to increase the coherence, continuity, and depth of the taught curriculum (NCTM, 2000; TIMMS, 1999). Yet, classroom evidence about how to improve mathematics education is still “dismayingly thin” (American Educator, 2005). The writing of specific tasks for classroom use remains difficult and of crucial significance to tackle (Laborde, 2001; Sfard, 2003). To address these issues and demonstrate a practical path to improving student learning in middle and high school years, it is important to carry out a project of instructional materials development. The project described in this article, Modeling in Middle and High School Mathematics, conceives of both students and their teachers as the “learners” to be supported in the acquisition of core mathematical content advocated by experts to be essential for re-envisioning middle and high school curriculum. This project is committed to supporting a reform curriculum, along with compatible instructional strategies, one that is driven by mathematical modeling as a way of effectively helping middle and high school students achieve higher academic performance in mathematics.

The project will be ground in the following theoretical perspectives: Cognitive Research on features of effective learning (Bransford, Brown, Rodney, & Cocking, 2000; McGowan, DeMarois, & Tall, 2000); Mathematical Problem Solving Theory (Polya, 1973; Schoenfeld, 1992; NCTM, 2000); Mathematical Modeling Theory (NCTM, 2000; Dossey et al, 2002; Lesh & Doerr, 2003); and Theory of Modeling with Interactive Technology (Kaput & Thompson, 1994; Nemirovsky & Monk, 2000). Specifically, the project will draw on the following modeling cycle perspective: A four-step modeling cycle that underlies solutions to a wide range of problem solving situations is: 1) description that establishes a mapping to the model world from the real (or imagined) world, 2) manipulation of the model in order to generate predictions or actions related to the original problem solving situations, 3) translation carrying relevant results back into the real (or imagined) world, and 4) verification concerning the usefulness of actions and predictions (Lesh & Doerr, 2003).

It will also be based on empirical research studies such as the research on students' modeling building and revision that shows impressive cross-sectional and longitudinal gains in achievement among the participating students, as well as changes in the participating teachers' beliefs and practices (Lehrer & Schauble, 2005); Hestenes' *Modeling Instruction in High School Physics* program, in which students of teachers who participate in Hestenes modeling workshops show

significant gains in science learning and in the acquisition of inquiry, reasoning, and problem-solving skills (US DOE, 2001); and Jiang & O'Brien's pilot study (2005), of which the main finding is that high school mathematics students taught by teachers who used the modeling approach outperformed those taught by teachers who used the existing curriculum and traditional approach in both basic skills and problem solving.

In summary, the goals of this proposed project include: 1) to develop mathematical modeling oriented instructional materials and learning environments that can be used to improve middle and high school students' mathematics learning; and 2) to develop mathematical modeling oriented professional development materials for middle and high school teachers. The modeling materials to be developed will meet the following criteria: using rich problem-solving tasks to reinforce and expand students' understanding of root concepts in algebra and geometry, having the potential to integrate required curriculum topics in a more coherent manner, emphasizing real-world mathematics and the use of technology, and being appropriate for a diverse group of students and teachers. The anticipated outcomes are improved learning performance of the students who use the modeling materials, improvement of their teachers' knowledge, positive changes of these teachers' teaching practice and beliefs, increased participation of students from underrepresented groups, and narrower achievement gaps between student populations.

2. Two Sets of Interrelated Materials

The anticipated products will be two sets of interrelated materials—Instructional Modules for Middle and high school Students (M-S) and Professional Development Modules (M-PD) for their teachers based on mathematical modeling and problem solving. These materials will be published as CD-ROMs, scholarly publications, and monographs.

The instructional materials to be developed are supplemental in nature. This is because most school districts have already selected their curricula for their schools. The teachers at those schools may not have enough flexibility to use a new comprehensive curriculum.

M-S will be for middle and high school students to use during the course of one school year, while M-PD will be for their teachers to use during intensive professional development activities. Student and teacher materials will differ in the number, depth and complexity of the mathematical topics and activities they contain. However, both sets of materials will flow from the same basic orientation toward the improvement of the teaching and learning of mathematics that informs this project: Focus on Core Algebra and Geometry Content (NCTM, 2000); the Use of Models and Model-Building (Lesh & Doerr, 2003), and the Use of Inquiry Activities (NCTM, 2000).

Sources for developing eight to ten modeling challenges for students at each grade level 6-12 will include, but not be limited to:

- HiMAP and GeoMAP Modules from the searchable database of the Consortium for Mathematics and its Applications (COMAP);
- Modules developed by Berry, Graham, and Sharpe at the Centre for Teaching Mathematics at the University of Plymouth, UK (Berry, Graham and Smith, 2005);
- Mathematical modeling activities developed by the author during his long time career of teaching children important and meaningful mathematics.

3. Detailed Descriptions of the M-S Materials

The guiding principles of designing the M-S materials are 1) the four-step modeling cycle perspective presented by Lesh and his colleagues that were cited in the Introduction section; 2) a

similar perspective presented by the NCTM (1989, p.328); 3) high expectations of middle and high school students by international standards (Schmidt, 2005); 4) a clear and limited content focus; and 5) the NCTM Core Curriculum idea.

Each modeling challenge will then have the following components:

1) Important mathematical ideas: Each modeling challenge will be designed to introduce or enhance students' understanding of two or more mathematical ideas that are connected and important to what students should learn at a specific grade level. For example, an instructional module (a modeling challenge) that will be given to ninth graders will focus on proportionality, triangle similarity, and the Pythagorean theorem.

2) Real world problem situation: A problem situation that could actually happen in real life will be presented first. The modeling challenge example mentioned in 1) will present the following Street Parking situation:

You are on the planning commission for your city, and plans are being made for the downtown shopping district revitalization. The streets are 60 feet wide, and an allowance must be made for both on-street parking and two-way traffic. Your job is to determine which method of parking – parallel or angle – will allow the most room for the parking of cars and still allow a two-way traffic flow.

3) Mathematical problem formulation: Students will be asked to formulate a mathematical problem that will produce a solution or solutions to the original situation. To do so, students will need to make certain simplifications or assumptions. For the Street Parking situation, for example, the following hints will be provided to the students: *Based on your experience or discussing with your peers, you may make assumptions related to the width of the roadway reserved for two-way traffic, the size of each parking space, parking space arrangements (road side parking or central parking), and whether there are other traffic or safety regulations.*

The students will then make their own simplifications or assumptions and formulate their own mathematical problems. In order to help them explore their own problems more effectively, this instructional module will present an example of the formulated problem. (Teachers should suggest that the students work on the example AFTER they have formulated their own problems.) The example is: *Consider the situation with the following conditions: Fifteen feet of roadway is needed for each lane of traffic. Parking spaces are to be 16 feet long and 10 feet wide, including the lines. Identical parking spaces are on two sides of the street. You may design parking for one city block (0.1 mile) and use that design for the entire shopping district.*

4) Investigation questions: To facilitate students' investigations for constructing a mathematical model for the formulated problem and solving the problem within the model, a set of thought-provoking questions will be given. The use of technology will be encouraged or required for the investigations to answer these questions. For the parking problem already formulated, the following questions will be presented for students to think about and explore:

i) Which method do you believe allows more cars to be parked?

ii) Draw a figure to represent the parallel parking situation. How many cars can be parked in this situation? *(The scaling or proportionality idea will be involved here.)*

iii) Try constructing a sketch like the one in the following figure *(see Figure 1 below)* with the Geometer's Sketchpad (GSP). What mathematical ideas must be used to construct such a sketch? *(A deeper understanding of the scaling or proportionality idea will be required here.)*

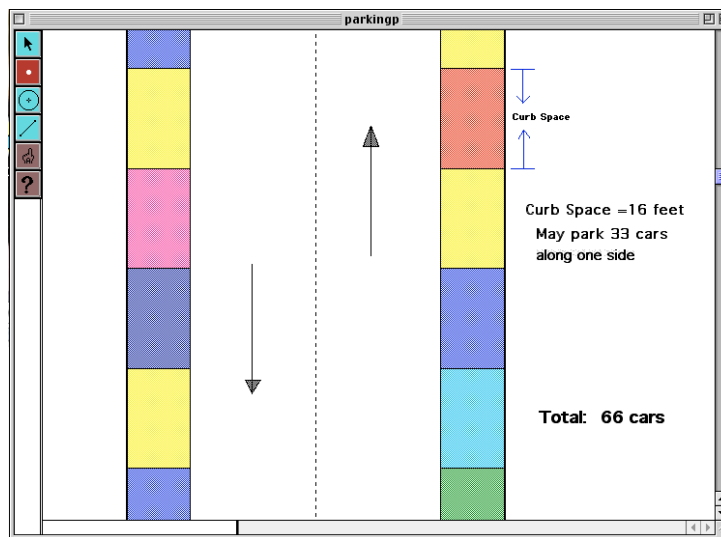


Figure 1. The parallel parking situation

iv) Try drawing a figure on a piece of paper or constructing a GSP sketch to simulate the angle-parking situation. *(Some students will be able to independently figure out a correct drawing or GSP construction. For those who have difficulties, further questions will be given.)*

v) If you get stuck, open the sketch *Parkinga.gsp* (which will be provided on a CD-ROM attached to the M-S materials) on the computer. What do you notice in the situation shown on your screen (see Figure 2 below)?

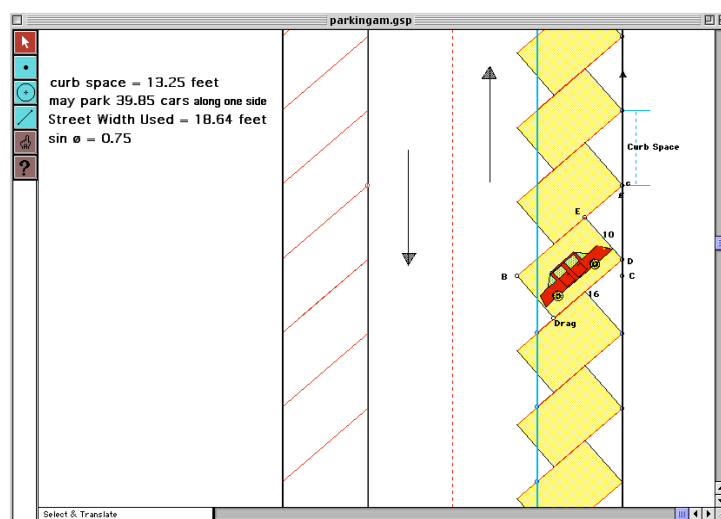


Figure 2. The angle parking situation (illegal)

vi) Is this parking situation on the screen good? Why or why not?

vii) How can you change the situation so that the parking is acceptable with the regulations?

viii) In order not to block the traffic, the left side of each parking space has to be “dragged down” so that the yellow rectangle does not intersect the lane of traffic (or at most “touches” the traffic lane at one point only). What do you observe when you do so?

ix) What does the visual feedback given by the following figure (see Figure 3 below) suggest? How many cars can be parked in this situation? *(Sound understanding of the scaling or proportionality idea will be required here.)*

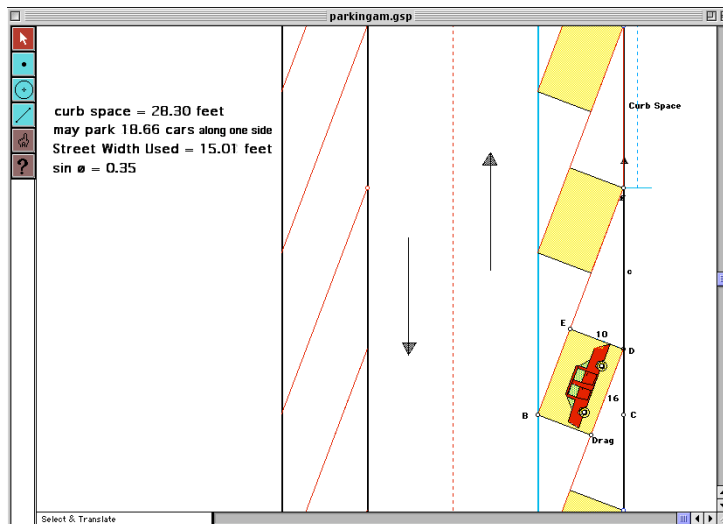


Figure 3. The angle parking situation (legal)

5) Interpretation of solution in original problem formulation: After the investigations, a mathematical model which is at least a pictorial/graphical or numerical representation will be constructed, and a solution within the model will be obtained. This will allow students to give interpretation of the solution for the formulated mathematical problem. In the parking problem example, the students will come up with a graphical model and get a solution suggested by the model with or without help from the materials-provided GSP sketch. Therefore, they can interpret the solution: *In the angle parking situation, the curb space is very long, resulting in the fact that much more space is wasted and fewer vehicles (36 cars) can be parked in one city block than in the parallel parking situation (66 cars).*

6) Validation in original real world situation: Students will carry the developed model and solution back into the original real world situation to test their correctness and usefulness. For the parking problem, if the original situation does happen in the real life, the results of the civic construction project will confirm the model/solution. Otherwise, a computer simulation will do the same thing. As a matter of fact, the GSP sketches (Figures 1-3) displayed above were constructed using geometry properties and proportional reasoning, and they themselves are valid computer simulations of the real word situation.

7) Further Discussion: If the constructed model is confirmed, then the original problem is considered solved until new information becomes available or assumptions change. Otherwise, students need to refine, revise, or reject the model. For the parking problem, even though the solution is confirmed, the graphical model so far lacks explanation or logical reasoning. Therefore, the model needs to be refined. *(Based on the NCTM Core Curriculum idea, some students may revisit this refining part later in the curriculum, consistent with their growing understanding of mathematics, while others may progress to work on this part during this instruction module.)*

The modeling challenge will help the students to do so through more investigation questions focusing on the mathematics relationships embedded in the graphical model. *(Due to limited space, these additional questions are not listed here.)*

Up till now, the students have developed a better model, and the solution will not only be based on a graphical model, but also a valid mathematical solution using triangle similarity and the Pythagorean theorem.

8) Extensions: This will be the final part of a modeling challenge. Mainly the students will be expected to work on two tasks. One is to explore different solutions by changing assumptions to see if a better model or solution can be achieved. The other is to push the investigations further or deeper. For the parking problem, the students will be asked to explore alternative solutions when one or two assumptions are changed. One possible change might be to change two-sides parking to central parking. Using GSP as an investigation tool, the students will be able to develop a new geometric model for both parallel and angle parking. The new model for angle parking is shown in Figure 4. Obviously, angle parking will allow more cars (76 cars) to park than the parallel parking (still 66 cars). This solution is opposite to the one mentioned above. (*Due to limited space, more alternative models and solutions are not discussed here.*) Then students will discuss among the different solutions, which one is the best in terms of both “allowing most cars to be parked” and “being the most realistic.”

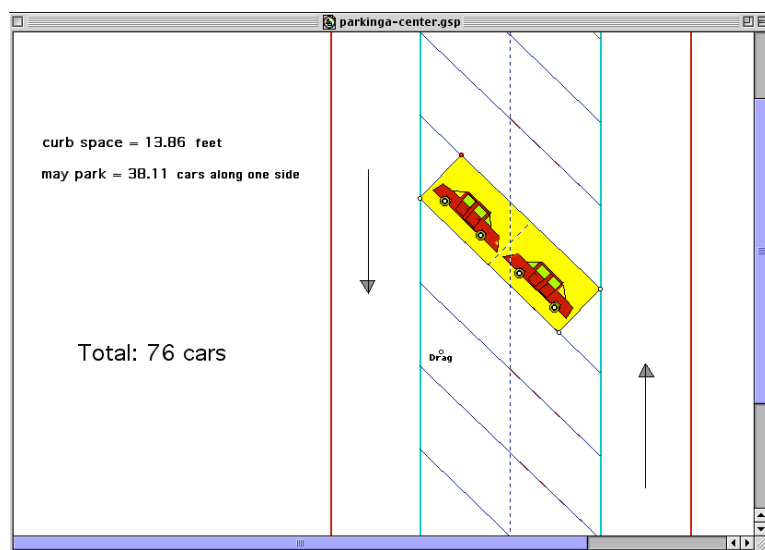


Figure 4. The situation under a different assumption

The students will also be asked to further their investigations by solving problems such as:

- 1) How wide should the street be so that angle parking allows more cars to park than the parallel parking if parking spaces are on the sides of the street (with other conditions the same)?
- 2) Taking everything we have done into consideration, which recommendation will you make to the city government regarding the downtown shopping district parking project?

4. Professional Development Resources

Different from some instructional materials development projects that provide teachers with basic training only, this project will offer an intensive workshop for teachers who will use the project developed materials in conjunction with existing curriculum in their classrooms. The rationale for this is the knowledge deficit of teachers compounding the weaknesses of U.S. curricula. Very few mathematics teachers (especially at the middle school level) today are resourceful enough to know the constant interplay between geometry and algebra. In New York schools, for example, there is a crying need for more intensive training of mathematics teachers (Posamentier, 2003).

The instructional materials developed for the intensive modeling workshop are the M-PD

materials, which will employ, initially, the same modeling challenges prepared for students, but will extend these into more advanced topics in algebra, geometry, and other strands to strengthen teachers' content knowledge. A set of mathematical modeling challenges will be designed specifically to engage teachers in making new connections among familiar content of the middle and high school curricula in use in their school districts. To provide teachers with experiences that allow them to deepen, extend and share their own knowledge and understanding of not only the content of mathematics, but also the ways in which students build mathematical ideas, and the pedagogical implications of powerful mathematical models, the M-PD materials will also include students' sample work of mathematical modeling for teachers to analyze, evaluate, and identify follow-up instructional activities. In addition, the teachers will be expected to share observations and insights that they made of their own students as they were working to solve challenging real life problems, identify the strengths and weaknesses of students' results, and assess the quality of students' work (Lesh & Doerr, 2003).

Another product to be produced is the Teaching Guides to accompany the student materials, which will include the mathematical expectations of each modeling challenge, detailed teaching suggestions, sample lesson plans, and sample solutions.

5. Content and Pedagogical Strategies

The project will address important individual and societal needs by providing constructivist pedagogy suggested by the NCTM (1989, 2000). A classroom using a modeling approach to learning typically employs a combination of student-centered, whole-class discussion of general ideas and questions; small-group explorations and discussion to encourage the investigation of alternative ways of thinking about and seeking a solution; and the justification of ideas and solutions by individual learners. These activities play an important role in developing students' conceptual understanding in mathematics and problem-solving abilities. The mathematical modeling approach strongly resembles what mathematicians and scientists do when they collaborate on a research task.

These materials will present the same content at different levels of abstraction through varying instructional strategies to account for potential differences among students. At each grade level 6-12, the materials will consist of a series of carefully developed, refined and structured problems through which challenging questions about physical nature and human society are explored by means of engaging mathematical games and models that convey core mathematical content of middle and high school curricula.

Mathematical modeling is a form of real-world problem solving. The modeling approach used in the materials to be developed will help students see the application of mathematics in sciences, engineering, technology, and other disciplines. Students involved in the modeling experiences will obtain a greater appreciation of the power of mathematics. This will play a positive role in preparing and motivating the students to continue to study sciences, engineering, technology, and mathematics at higher-grade levels.

An important feature of this project will be the emphasis on the use of technology. The materials to be developed will involve significant use of various technologies including general tools such as word processing, paint and draw programs, and spreadsheets; the Internet; application software such as the Geometer's Sketchpad and Fathom; and graphing calculators such as TI-84. As indicated by the NCTM, "Electronic technologies—calculators and computers—are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately.

They can support investigation by students in every area of mathematics, including geometry, statistics, algebra, measurement, and number. When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving” (2000, p. 24). Because mathematical modeling is based on applying mathematics in the real world situations, the use of technology is natural and sometimes indispensable in various mathematical modeling situations. For example, many modeling situations involve scatter-plotting the given data, and using the plotted graphs and curve fitting techniques to obtain models (usually functions) to make predictions. In these situations, the use of suitable curve-fitting technology (computer software or graphing calculators) becomes crucial. In the Street Parking activity presented in a previous section of this article, people can also see the role of technology in the mathematical modeling process clearly: the dynamic, graphical representation of the problem situation can effectively help students realize and correct their possible misunderstandings, and students’ further investigation with the dynamic geometry software not only allows them to build good models for solving the problem, but also facilitate their development of mathematical proofs for the correctness of the models. Research reveals that two equalizers that work well with a wide range of students are the use of technology and the incorporation of applications and real problem solving into the mainstream of the curriculum (Usiskin, 1993). These two aspects are the most significant characteristics of our modeling challenges. They will make the curriculum taught more relevant, motivational, comprehensive, and hence will offer greater opportunities for all students including underrepresented students to engage in meaningful mathematics learning.

6. Rationale

As previously pointed out, problems with K-12 mathematics education in the U.S. are extensively documented. Curriculum content—what is taught and what is learned—is the heart of the problem. Research characterizes K-12 mathematics curricula in the U.S. as “repetitive,” “unfocused,” “unchallenging,” and “incoherent” by international standards, and points to the acute weaknesses of middle and high school curricula where up to 75% of the curriculum is sheer repetition. By the end of eighth grade, while children in the top achieving countries have mostly completed mathematics equivalent to U.S. high school courses in Algebra 1 and Geometry, most U.S. students are destined for the most part to continue the study of arithmetic (Schmidt, 2005). According to the American Association for the Advancement of Science, middle and high school mathematics texts tend to “bury” key concepts and they rarely “model the use of scientific knowledge so that students [can] apply what they learned in everyday situations”. According to NAEP 2005 Report, 70% of the U.S. eighth graders show limited or less than limited skill in communicating mathematically.

To have a coherent, focused, and demanding curriculum for all children, incorporating mathematical modeling experience in the curriculum is a solution. As a matter of fact, since 1975, many national conferences and committees have been advocating an increased emphasis on mathematical modeling in school curriculum (Conference Board of the Mathematical Sciences, 1975, 1984; National Research Council, 1989; NCTM, 1980, 1989, 2000). However, despite these repeated recommendations and many research findings confirming the importance of mathematical modeling, not enough effort has been expended in designing mathematical modeling-based instructional materials for students and preparing teachers to use mathematical modeling techniques and situations in their classrooms, particularly at the middle and high school levels.

Some curricula, especially reform curricula such as *Connected Mathematics*, have paid close attention to real world applications of mathematics. However, just as “word problems are not

automatic substitutes for applications” (Usiskin, 1993, p.20), applications are not automatic substitutes for mathematical modeling. Based on the definitions given by researchers (e.g., Lesh & Doerr, 2003) and NCTM Standards documents (1989, 2000) that we have discussed in this proposal, mathematical modeling is a multi-step and multi-cycle process involving activities such as problem formulation, model exploration, model building, model manipulation (to find a solution or solutions), solution interpretation, and solution validation/verification.

In order to formulate a mathematics problem from a real-world situation, simplifications or assumptions need to be made. For different simplifications or assumptions, models developed and solutions obtained are usually different. Because of the requirement for high expectations and worthwhile opportunities for all students (NCTM, 2000; Schmidt, 2005), the problem situations that we give students to practice mathematical modeling should be nontrivial, demanding ones. In addition, solution validation/verification is to go back to the original situation and see if the results of model building and manipulation make sense. A person might go through a revision process several times before he or she is done making a good mathematical model, as mathematical modeling is a cyclic process in nature. In this process, students need to work with a variety of mathematical concepts, processes, and relationships, and connect their understanding of specific content to the modeling situation.

Compared to these characteristics uniquely belonging to mathematical modeling, many application problems in the current curricula are not mathematical modeling or not at the mathematical modeling level. This proposed project is an effort to enhance the current curricula, aimed at providing opportunities for students to develop abilities of applying what they learn to creatively solve various real world problems, achieve real understanding of important mathematics required at their grade level, and accumulate skills that will help them in just about any career they might choose in the future.

7. Conclusion

The data from the national and international assessments and studies such as NAEP and TIMSS demonstrate the importance of a coherent and demanding mathematics curriculum. To help contribute to the development of such a curriculum for the U.S. middle and high school students, this article has discussed the connections between using mathematical modeling based materials and improvement in mathematics education and attempted to propose a project of developing mathematical modeling oriented instructional materials for both middle and high school students and their teachers. Built on supporting theoretical perspectives and empirical research, this project will help determine if mathematical modeling approach leads to improved mathematics achievement across ethnic, racial, and socioeconomic groups. The project tackles one of the weakest parts of K-12 mathematics education with a plan for strengthening curriculum and teacher professional development simultaneously, based on realistic opportunities for collaborative partnership between universities and school districts that have a large population of underrepresented minority and low SES students. Success of the project would pave the way to its extension more broadly to urban middle and high schools nationwide.

References

American Educator Editor. (2005). Mathematical knowledge for teaching: A research review. *American Educator* Vol. 29, No. 3, pp. 18-19.

Berry J., Graham E. & Smith A. (2005). Classifying student’s calculator strategies. *The International Journal for Technology in Mathematics Education*, 12, No. 1, 15-31.

Bransford, J. D., Brown, A. L. & Cocking, R. R. (Eds.) (2000). *How People Learn*. Washington, DC: National Academy Press.

Conference Board of the Mathematical Sciences and National Advisory Committee on Mathematical Education. (1975). *Overview and analysis of school mathematics grades K–12*. Washington, DC: Conference Board of the Mathematical Sciences.

Conference Board of the Mathematical Sciences. (1984). *Graduate education in transition*. Washington, DC: Conference Board of the Mathematical Sciences.

Dossey, J., Charles, R. I., Leinwand, S. J., Seeley, C. L. (2002). *Middle and high school math: Course 2*. Pearson Prentice Hall.

Jiang, Z., & O'Brien, G. (2005). *A pilot study on South Florida's teacher education addressing the needs of at-risk students*. (unpublished).

Kaput, J., & Thompson, P. (1994). The first 25 years of research on technology use in mathematics education. *Journal for Research in Mathematics Education, 25th Anniversary Edition*.

Laborde, C. (January 2001). *Teaching Styles*. A presentation at a teacher workshop at Dubach High School, Dubach.

Lehrer, R. & Schauble, L. (2005). Developing modeling and argument in the elementary grades. In T. Romberg, T. Carpenter, & F. Dremock (Eds.), *Understanding mathematics and science matters*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R. & Doerr, M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & M. Doerr (Eds.), *Beyond Constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

McGowen, M., DeMarois, P., & Tall, D. (2000). Using the function machine as a cognitive root. In *Proceedings of PME-NA*, Arizona.

National Council of Teachers of Mathematics. (1980). *An agenda for action*. Reston, VA: The Council.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: The Council, 1989.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: The Council.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, D.C.: National Academy Press.

Nemirovsky, R., & Monk, S. (2000). If you look at it the other way: An exploration into the nature of symbolizing. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design*. Mahwah, NJ: Lawrence Erlbaum Associates.

Polya, G. (1973). *How to solve it*. Princeton, NJ: Princeton University Press.

Posamentier, A. S. (2003). Educating math teachers. In *Education Update Online*. <http://www.educationupdate.com/archives/2003/apr03/issue/col-educating-math.html>

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grows (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Schmidt, W. (2005). The Role of Curriculum. *The Professional Journal of the American Federation of Teachers, Vol. 29, No.3*, (PP. 11-13). Washington, DC: American Educator.

Sfard, A. (2003). Balancing the unbalanceable: The NCTM Standards in the light of theories of learning mathematics. In J. Kilpatrick, G. Martin, & D. Schifter (Eds.), *A research companion for NCTM Standards*. Reston, VA: NCTM.

TIMSS. (1999). www.timss.org/timss1999.html

Usiskin, Z. (1993). If everybody counts, why do so few survive? In G. Cuevas & M. Driscoll (Eds.), *Reaching all students with mathematics*. Reston, VA: NCTM.

U. S. Department of Education. (2001). <http://www.whitehouse.gov/infocus/education>.