Cabri 3D: some possibilities

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Abstract: Cabri 3D is a relatively new software which has great potential in teaching and learning and in the development of new mathematics. An obvious use of Cabri 3D is in the three-dimensional geometry of polyhedra, planes, cones, prisms, etc. However, it may also be used to facilitate visualization and as a mediator between the world as experienced and the abstract world of 2D geometry. It also enables the modeling of both architectural and mathematical structures and of different types of objects in motion. This paper will explore a number of possibilities and introduce an approach based on an integration of text, hypertext, both static and dynamic "pictures" and interactive demos.

Introduction

My first response to a prototype of Cabri 3D, demonstrated in 2001,was "so what?" Oldknow (2005) suggests that this might be a common response "It is possible to view…*Cabri 3D* to be just for fun and of no curricular relevance!"

After experimenting with Cabri 3D for the past two years I am now convinced that it is an important software with the potential to radically alter our perceptions of geometry, open up new mathematics and provide engaging mathematical environments for students.

This paper will illustrate a number of possibilities with Cabri 3D and will introduce a particular approach to its instrumental genesis.

Three-Dimensional Geometry

In considering Cabri 3D in 3D geometry it is useful to focus on both the fundamental similarity between Cabri 3D and 2D interactive geometries and also the differences between these softwares.

An important view of mathematics is as a science dealing with variable objects (Laborde, 2005, p. 23). In algebra such an object might be an identity such as $(x – 3)(x + 5) = x^2 + 2x - 15$, which remains true as the value of x varies. A geometry theorem, which states that certain geometrical properties (such as the relationship between the squares on the sides of a right-angled triangle) remain invariant as a figure varies are comparable to such identities. Interactive geometry software, such as Cabri 2+, Geometer's Sketchpad and now Cabri 3D is hence of particular importance in that students, in being enabled to manipulate tangible variable objects, such as right-angled triangles, are introduced to this essential feature of mathematics.

As Cabri 3D shares this feature with the well-researched 2D interactive geometries, much of the research evidence concerning 2D interactive geometries is likely to generalize. For example, Mariotti (2006, p. 193), in a summary of research on proof, has found that interactive geometry environments have in general been successful in enabling students to link informal explanations with formal proof. Cabri 3D will hopefully enable the same linking with regard to results in 3D geometry. Laborde (2005) has discussed the problem of finding a tetrahedron in which the altitudes intersect using Cabri 3D and has shown that soft construction, in which certain properties are constructed visually rather than geometrically, can enable both the solution of the problem and an understanding of why the solution works.

Cabri 3D also has some important differences from 2D interactive geometry, which can act as both affordances and constraints. One difference is that many of the features of Sketchpad or Cabri 2+ are still in the process of being developed. Measurement is expected to be introduced in autumn 2006. Other features not yet present are macros and loci. However, some of the limitations have also been enabling. Two pedagogical models concerning volume and the cross-product of two vectors are shown later in this paper. The absence of measurement made the creation of these files an interesting mathematical challenge rather than just a useful task.

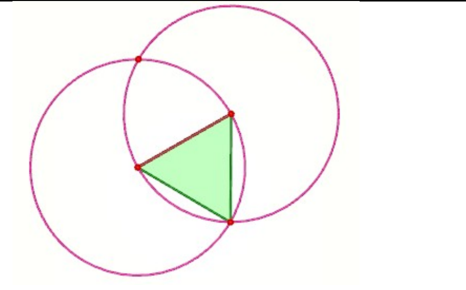
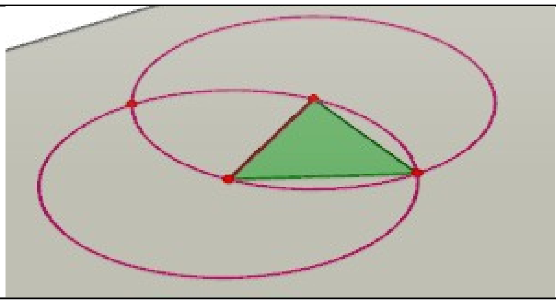
Cabri 3D also has features not shared with 2D interactive geometries, such as the ability to create polyhedra, cut these polyhedra to form new polyhedra, and to create the nets of polyhedra. These have immediate relevance to the learning of 3D geometry. Whilst various other 3D geometry softwares such as Poly share some of these features, Cabri 3D is unique in providing an environment in which construction, transformation and manipulation are also possible, enabling richer insights into 3D geometry.

A potential issue concerning the difference between Cabri 3D and 2D geometry software is that there is an inevitable loss of information in any 2D representation of 3D space, as any point on the screen represents a line in space (Parzysz, 1988). This is overcome to some extent by the ability to change the viewing angle, but creating or dragging any unconstrained object may have unexpected results: what looks like a small difference from one viewing angle may create a large change . Hence free "play" with object creation may be less satisfying than in 2D. On the other hand, geometric properties have greater importance: the student is forced to continually consider the geometric relationship between objects, as any apparent spatial relationship is unlikely to hold true.

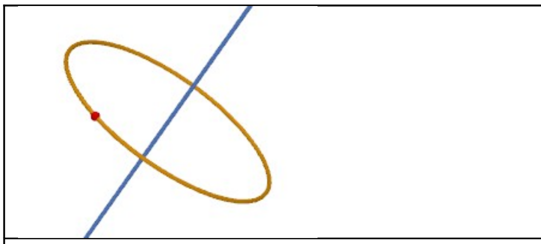
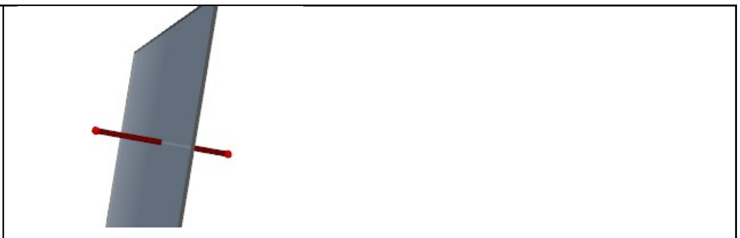
Another consequence of this limitation is that it is not possible to easily infer the properties of objects on the basis of spatio-visual information, as found by Accascina and Rogora (2005). Students using Cabri 3D were unable to decide whether all parallelogram cross-sections of a cube were rectangles simply by viewing the screen. On the other hand, the consequent uncertainty may act as incentive for proof (Hadas, Hershkowitz &Schwarz, 2000).

Cabri 3D in 2D geometry

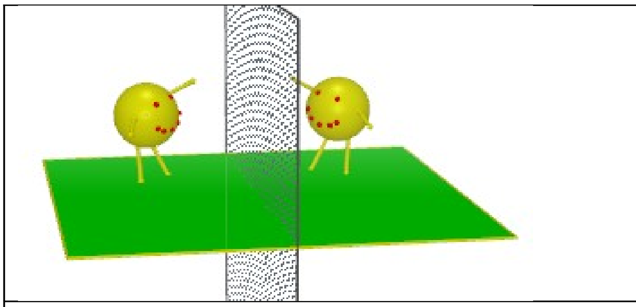
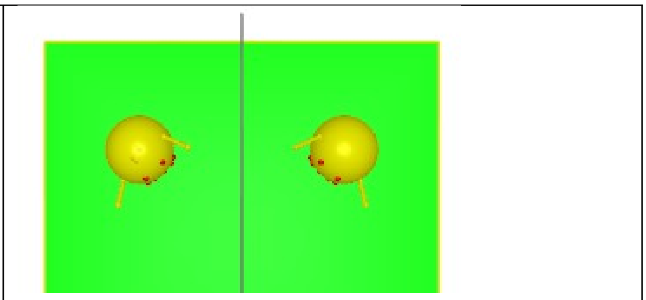
Constructions in 2D geometry may be performed on any plane in Cabri 3D, and hence can be easily viewed from different directions. This may enable students to more readily distinguish between incidental spatio-graphical properties such as the position or orientation of an object and the necessary links between its spatio-graphical properties which arise from the way the object was constructed. This distinction is critical for understanding the nature of these links (Laborde, 2005). This is illustrated by the two views of the construction of an equilateral triangle below.

	
Looking directly down on the base plane. The orientation of this figure is readily changed.	*A view from an oblique angle: an accurate construction which has given rise to an "inaccurate" diagram, necessitating separation of the visual and the geometrical for its interpretation.*

Cabri 3D also provides a link between 2D and 3D geometry. Familiar 2D objects may need to be constructed in new ways, or change substantially in 3D, as illustrated below:

	
A circle is defined by a point and an axis.	*The perpendicular bisector of a segment is a plane.*

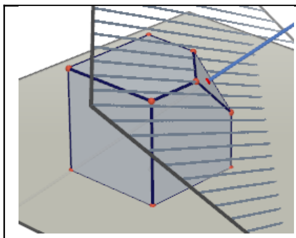
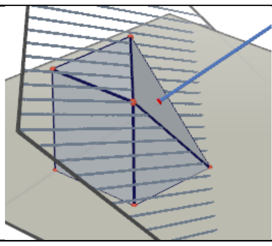
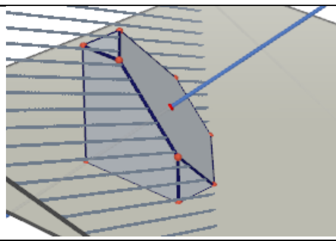
Rather than 3D space being considered an extension of 2D space, 2D objects may hence be seen as embedded in 3D space. Cabri 3D may thus mediate between the 3D world of experience and the more abstract world of 2D geometry. Here is an example which uses a character called Claude to mediate the mathematical meaning of reflection:

	
The Claude "behind" the mirror is a reflection.	*Claude, reflection, and mirror seen from above: a typical reflection in 2D geometry.*

Visualisation

The ability to visualize is an important aspect of doing mathematics and does not develop through ordinary experience. Hence a major issue is the development of "effective pedagogy that can enhance the use and power of visualization in mathematics education" (Presmeg, 2006, p. 227). Research evidence shows that the use of dynamic computer software can form part of such effective pedagogy. (Presmeg, 2006, p. 220).

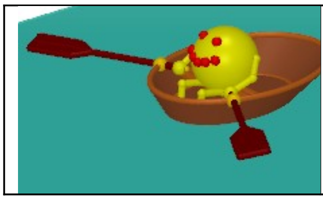
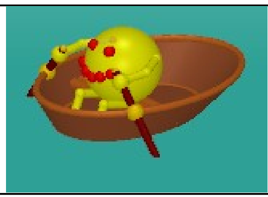
Cabri 3D may facilitate 3D visualization through allowing students to manipulate structures which are not limited by gravity or solidity and which can be given varying degrees of visibility. In a computer environment, such manipulation involves an explicit awareness of the nature of actions such as rotation upon objects (Gutiérrez, 1996). Here are some questions using Cabri 3D which are designed to require student visualization:

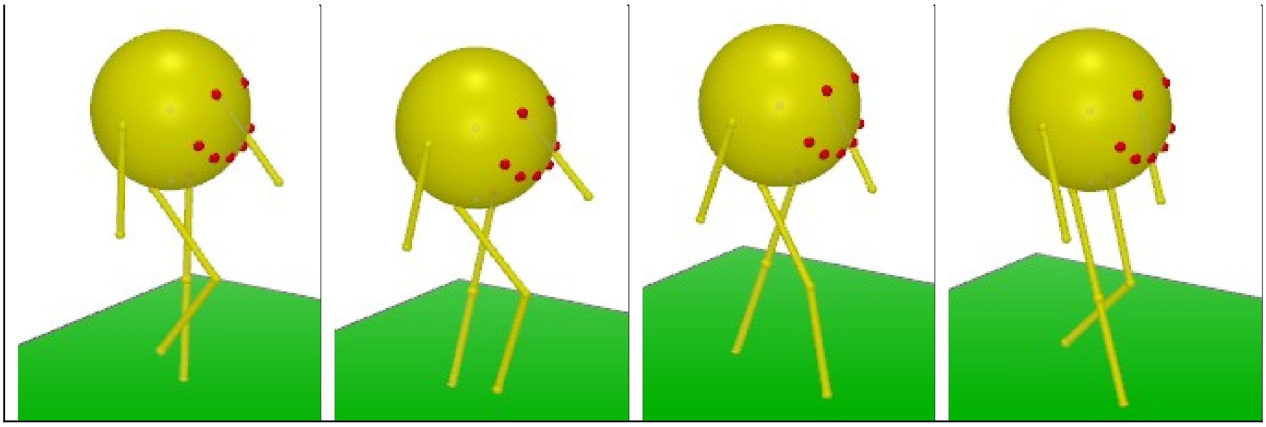
		
What shape is this cross-section?	*What happens to the cross-section as the plane moves towards the bottom vertex?*	*What characterizes this hexagon? Is it possible to get a regular hexagon as a cross-section?*

Physical Modelling

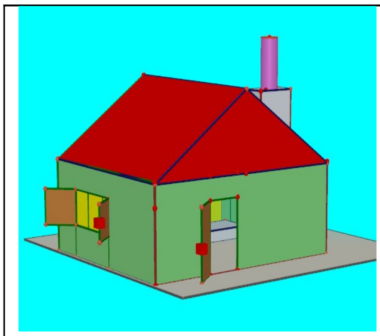
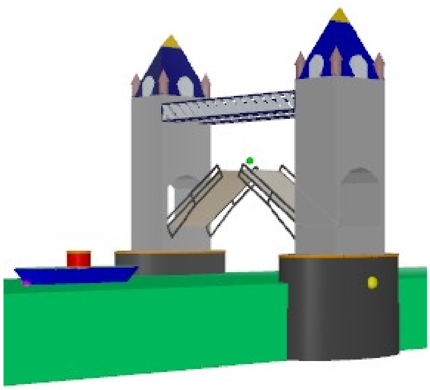
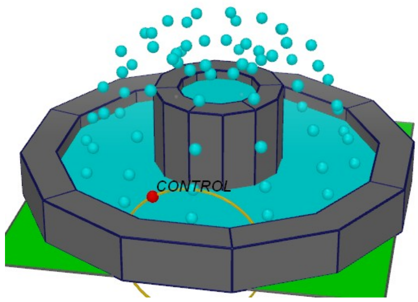
One of the reasons for the early importance of Logo was its connection with bodily motion: the turtle represented an object turning in the physical world. The lack of correspondence between the geometry of Logo and the Euclidean geometry of the school curriculum has been problematic, however (Laborde, Kynigos, Hollebrands, & Strässer, 2006).

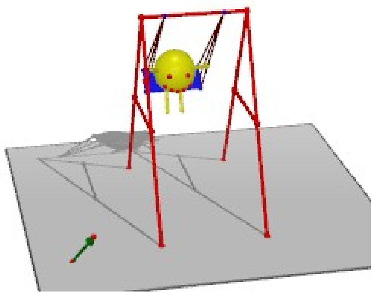
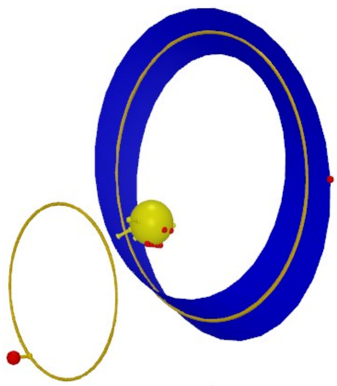
Cabri 3D can connect bodily motion and Euclidean geometry, however: an awareness of both is required to create files such as those shown below, in which Claude rows a boat or walks.



A similar explicit awareness of the geometric relationships embedded in physical structures, both static and dynamic, is required to create the objects that follow:

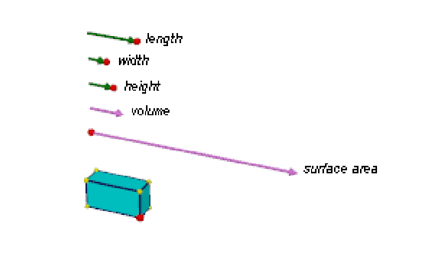
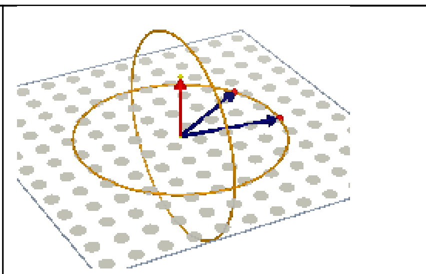
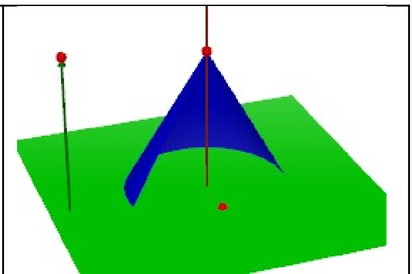
		
Little House by Jean-Jacques Dahan	*Tower Bridge by Adrian Oldknow (file available at www.counton.org/cabri/*	*The water in the fountain flows as the control point is moved or animated*

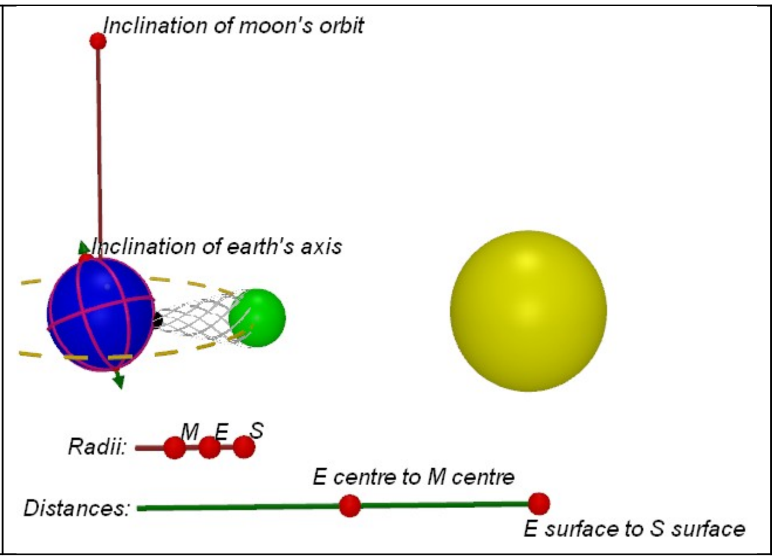
The swing swings and the sun direction can be changed.	*The carousel rotates and the "seats" go up and down.*	*Claude slides around the mobius strip.*

Pedagogical Models

These models, which cannot be entirely reproduced in the physical world, are designed to illustrate mathematical dependencies:

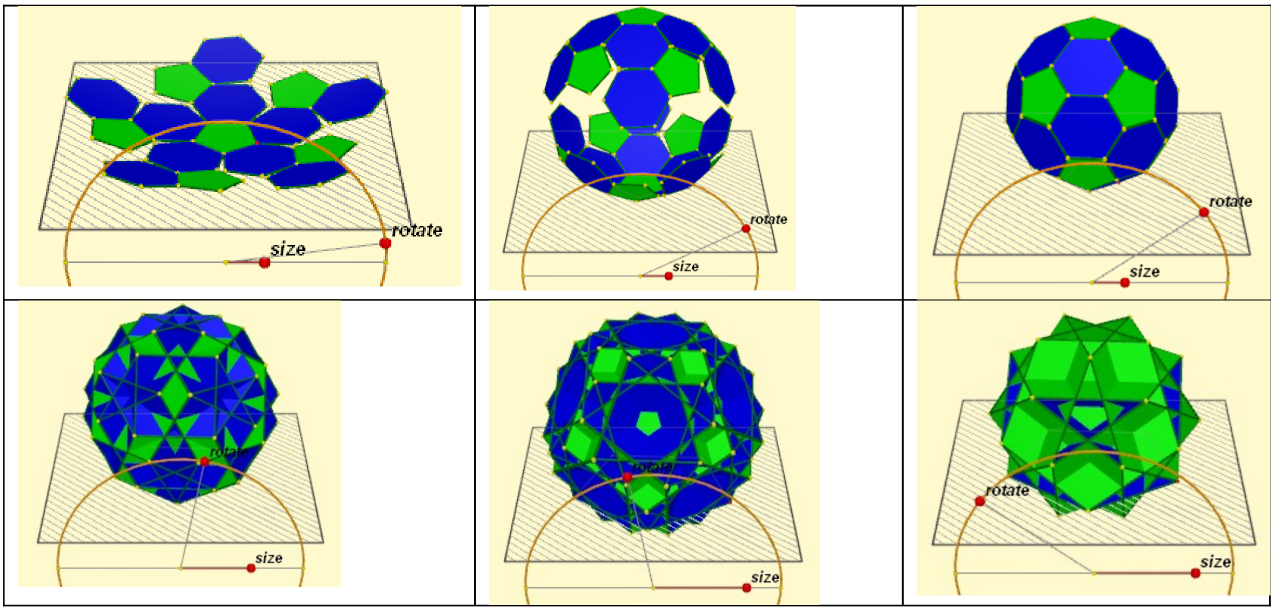
		
Effect of changing length, width or height on volume and surface area.	*The cross product of two unit vectors as the angle between them varies.*	*A cone of variable height and radius can be transformed into a sector.*

Models for science teaching can also be created, such as the model of the total eclipse shown below:

The "earth" and "moon" are animated, with the "earth" rotating around its axis and the "moon" orbiting the earth every 28 revolutions. The black sphere shows the location of the intersection between the earth and the moon's shadow and hence the location of the region where a total eclipse of the sun occurs.	

New Mathematics

As well as enabling new results in research mathematics (e.g. Oldknow (2005) generalized the Soddy line of a triangle to a tetrahedron), objects may be created which give rise to areas of new mathematical exploration which are both attractive and accessible to students. An example is the "net" of a truncated icosahedron in which the dihedral angle between connected polygons is progressively being decreased (Mackrell, 2005a):



The unexpected emergence of the final three objects, with high degrees of symmetry, gives rise to the question "why", which leads to further investigation and proof.

Problem:

Given the potential of Cabri 3D, how do we engage learners? In the early years of the use of technology, it was assumed that learning would emerge simply from the interactions between the student and the machine. It is now recognized, however, that the choice of task and learning environment is crucial (Laborde, Kynigos, Hollebrands, & Strässer, 2006, p. 279). An initial problem is that of instrumental genesis: how can we provide a learning environment in which Cabri 3D progresses from being an artefact to becoming an instrument (Rabardel, 2002) which can be used in problem-solving or design where appropriate and laid aside in favour of other instruments when inappropriate?

Laborde, Kynigos, Hollebrands, & Strässer (2006, p. 280) recognize that there is an intrinsic link between mathematical knowledge and knowledge about how to use a tool and that hence developing the ability to use a tool may also involve developing mathematical knowledge. This accords with my experience with teacher education students engaged in learning how to use Cabri or GSP in Ontario and in the UK. Many learners have very little geometrical awareness and hence cannot easily distinguish between a geometry problem and a problem of not knowing how to use a particular tool. For example, a student, unable to create a sequence of equally spaced points on a line commented that this was due to not knowing how to use the "equidistant" tool.

Hence, for many learners, teachers as well as students, the process of instrumental genesis must also include enhancing geometrical understanding. This is an important justification for using interactive geometry software in a classroom: with relatively straightforward interfaces, learning how to use the software is mainly about coming to grips with the underlying geometry. In the process the software will become the geometry in some sense for the student (Mariotti, 2002), and its particular features may have a profound effect on geometrical thinking. A concept image of "perpendicular bisector" based on the use of a straight edge and compass will differ from that of students creating a concept image through using the Cabri 3D "perpendicular bisector" tool.

Possibilities:

An early effort with multi-leveled text was a resource consisting of three documents for learning how to construct a kaleidoscope using GSP or Cabri. The first document gave a framework for the task and asked specific questions and suggested extensions. The other two documents showed the process in detail, using screenshots, one with Cabri and the other with GSP. Learners were encouraged to use the documents as they felt comfortable: the confident could use the framework as a set of challenges to meet whereas those lacking in confidence could follow the process in detail. This has had a positive response in a number of workshops for teachers. It allows learners to work at their own pace, choose their level of challenge and create a mathematically and aesthetically pleasing object.

An introduction to Cabri 3D was designed in a similar manner, consisting of activities now posed at three levels of challenge with details communicated by screenshots, followed by extension questions (Mackrell, 2005b). One issue, however, is that all hints and details are visible and it is hence hard to ignore the details. Another is that younger learners may not feel encouraged to continue exploration. Using similar materials showing how to create Claude on the swing, a mathematically able sixteen year old felt challenged, but was able to create the figure. A 12 year old with extensive experience with Cabri 2+ was able to follow the materials and create the figure, but showed no interest in further work with Cabri 3D.

I have recently been designing interactive demos using the software TurboDemo. These show brief movies of the steps involved in solving a problem, but also ask the viewer to anticipate what might happen next, to consider mathematical questions and to engage in extension activities. The viewer is in control, choosing whether to proceed at the end of a step or to view the step again. Combining this with the idea of multi-layered text, I am now in the process of writing an HTML resource to introduce fold-up polygons such as the truncated icosahedron shown above. This includes hypertext (with links to a glossary), popups for hints, pictures, embedded Cabri 3D files and interactive demos. Part of this resource may be found at http://educ.queensu.ca/~mackrelk .

In trying out this resource, it has become clear that 12 year olds can easily follow a demo and create a fold-up dodecahedron. A major question, however, is whether students are learning a sequence of moves to follow or are learning about the rotation which is the purpose of these moves, particularly as little interaction with text is required when following a demo. This will be the subject of further research.

In conclusion, Cabri 3D has great potential, but, as with any interactive geometry software, instrumental genesis is problematic due to lack of geometric understanding on the part of both teachers and students. New approaches are needed to overcome these difficulties and a web-based approach is being developed.

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