Technology in Mathematics: A Course on Doing Mathematics with the Natural Inclusion of Technologies (Computer and Calculator)

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Abstract: The preparation of our future mathematics teachers for appropriately using technology is one of the most important issues facing our teacher education programs. In response to this immediate need, a course was recently developed for pre-service secondary level mathematics teachers at an internationally diverse American university. This paper explains the three-fold purpose of the course. Objectives for the students included: 1) learning how computer software and calculators can be used as authentic tools in solving mathematical problems, 2) to be able to work problems that may not have been accessible without the use of technology, and 3) to review secondary and tertiary level mathematics concepts within a new technology-based context.
The emphasis of the course is not on technology, rather on doing mathematics, where use of technology meaningfully assists in understanding and solving the task by visual, graphical, or numerical means. Hence, specific technologies are introduced within the context of doing mathematics. Our present course includes problems from algebra, trigonometry, probability, statistics, geometry, and calculus (single variable and multivariable).
Many students who have limited experience using technology in mathematics believe its main purpose is to check answers, or to be used after a concept is learned. During the course, the students realize there are many more valid uses for technology in mathematics. Guidelines to follow when selecting and developing mathematics activities that integrate technology, are outlined. Examples of good mathematics problems that naturally lend themselves to the use of technology are examined.

Introduction

As the twenty-first century begins to unfold, technology is slowly becoming more visible in mathematics classrooms worldwide at all levels. The National Council of Teachers of Mathematics (see [8]) and in a position paper (see [9]), stated, "Technology is an essential tool in teaching and learning mathematics effectively; it extends the mathematics that can be taught and enhances students' learning." In addition to increased U.S. interest in technology in the mathematics classroom, many other countries are also exploring appropriate uses for various technologies in the teaching and learning of mathematics at the secondary and tertiary levels. (see [5], [12], and [6]) The appropriate use of different technologies in teaching and learning mathematics has become an important international issue.

In [1] it was concluded that the advent of different types of technology would inevitably influence the teaching and learning of mathematics. And, that a major challenge to mathematics educators is how to use these technologies to enhance their competencies and increase their effectiveness as teachers and to improve the math understanding and ability of the students. In [1] the authors continued, “One indisputable fact is that the impact of technology varies when technological tools vary.”

Research and experience has shown there are several key factors that influence when and how technology is use in the mathematics classroom. First, the institutional value which the technology is given influences the degree to which students are willing to apply themselves in learning the technical skills necessary to work with the technology. Second, the use of multiple representations has been shown to increase students' conceptual understanding and also provides them with alternative methods in solving problems. Finally, it has been found that students need to be guided in judicious use of the technology (see [10]). This involves teaching students to discriminate in their use of technology, that is, when a technology tool will be of great assistance in working a problem and when paper and pencil methods would be the better approach.

The best way for students to learn to use technology in solving problems would be for them to have hands-on experience with different technologies in mathematics courses. However, the type of technologies used in college level mathematics courses may differ from that appropriate for elementary and secondary level students. Hence, there is a need to address the use of technology in doing mathematics within teacher education courses.

There are a number of ways to incorporate technology into teacher education. In [3] various approaches were categorized according to the primary user of the technology or controller of the technology that being, teacher educator, teacher, or student. In [3] it was concluded the most direct and effective way to use technology to bring about enhanced student learning of mathematics is to prepare pre-service teachers (PSTs) to incorporate into their teaching an array of activities that engage students in mathematics thinking facilitated by technology tools.

Guidelines for Developing the Technology in Mathematics Course

Based on research findings and personal experience, the following guidelines for development of technology-based mathematics activities were established for the new course and are recommended to others when developing a course of this nature. First, features of a technology ought to be introduced and illustrated within the context of meaningful mathematics-based tasks. One should keep in mind, the use of technology in mathematics teaching is not for the purpose of teaching about technology, rather for the purpose of enhancing mathematics teaching and learning through the use of technology. Second, care must be taken not to use technology to teach the same mathematical topics in fundamentally the same ways that could be taught without technology. This belies the usefulness of the technology and does not strengthen students learning of mathematics. Technology can actually be a hindrance to learning if used to do mathematical tasks that can be done just as easily without the technology. Therefore, the mathematics tasks should take advantage of the capabilities of the technology by extending beyond or significantly enhancing what could be done without the technology (see [3]).

Another related guideline is to incorporate appropriate pedagogy when addressing worthwhile math tasks. Activities should support sound mathematical curricular goals and should not be developed merely because technology makes them possible. The activities developed should facilitate conceptual development, reasoning, explorations, and problem solving within the goals of the mathematics curriculum. Mathematics tasks involving the assistance of technology can be developed to lead students to better comprehend the necessity for mathematical "proof' and "rigor".

The fourth guideline is to utilize technology in connecting mathematical topics that may not have been previously possible or too unwieldy to consider. When the activities were being developed for the new course, mathematical connections were in two ways, by interconnecting mathematics topics and by connecting mathematics to real-world phenomena. Technology "blurs" some of the artificial separations among some topics in algebra, geometry, and data analysis by allowing students to use ideas from one area of mathematics to better understand another area of mathematics" (see [8] p.26). The final suggested guideline is that whenever possible, the technology-based activities should incorporate multiple representations of mathematical topics. The use of technology can make the inclusion of a number of mathematics applications more practical and allow the user to bring together multiple representations of math topics. Research shows that many students have difficulty connecting the verbal, graphical, numerical, and algebraic representations of mathematical functions (see [4] and [7]). Graphing calculators and computer software are capable of providing multiple representations of mathematical concepts.

Creating the Course

Recognizing the importance for pre-service secondary mathematics teachers to develop knowledge and skills in teaching and learning mathematics utilizing technology, the Mathematics Department at BYU-Hawaii recently developed a course to introduce a number of available technologies to the students. The three-fold purpose established for the course was to have students: 1) actively participate in learning how computer software and calculators can be used as authentic tools in solving mathematical problems, 2) be able to work abstract or real world problems that may not have been accessible without the use of technology, and 3) review many secondary and tertiary level mathematics concepts within a new technology-based context.

Given the time constraints of a three-credit course, one challenge in creating the curriculum came with deciding which of the many technologies should be included in the course. Another challenge/opportunity came in the fact that a textbook for such a course does not currently exist. Thus, the technology-based mathematics tasks would either need to be created from scratch or located from various resources. When developing the new course, emphasis was placed on obtaining student engagement with the math task and providing hands-on experience with the technology.

Most of the math problems selected for the course were chosen because the concepts have historically been difficult for students to understand. The topics included probability, polygon and mid-polygon area and perimeter relationships, conic sections, optimization applications with and without calculus, recursion, affects of coefficients on graphs of functions, affects on mean and standard deviation when data are altered, matrix applications, derivative and tangent line relationship, antiderivatives, 3D graphing with applications, and modeling. Since the students were pre-service mathematics teachers, tasks on learning to create professional looking mathematics documents such as exams and being able to develop a grading program were included. The students were also asked to critique each assignment. The culminating experience for the students came in the form of a project where they were to create their own set of technology-based mathematics activities and determine the appropriate type of technology to be used in working the problems. The project included detailed student worksheets, step by step use of technology, and a teacher's guide.

After reviewing how technology has been successfully used as a teaching and learning tool in doing mathematics, it was determined the minimal type of technology that should be included for the mathematics activities in the course would be graphing calculators, spreadsheet software, a software program for writing mathematics symbols and text, Computer Algebra System (CAS) software, and dynamic geometry software.

A Sampling of Course Tasks

Use of technology as a tool, in solving selected mathematical tasks, took four general forms in the course. Some of the tasks included "ready to use" tutorials or sketches created with the technology to assist students in doing the problem. Other activities gave students step-by-step instructions for the technology to create a visual, numerical, or symbolic representation of the math problem to be explored. Once a particular technology had been used a few times, then math tasks were assigned with only a few general guidelines or suggestions on how the technology could assist in solving the math task. For a few assignments, students were given the math problem without receiving guidance on an appropriate technology to use. These problems were challenging enough that students would want to use technology to aid in completing the assignment. These types of assignments helped the students become more technology *savvy* by requiring them to consider the different features and advantages of the various technologies.

Example of Math Task with "Ready to Use" Technology

The following problem was the first one assign in the course. The objectives were to illustrate the benefits of doing the problem utilizing a technology tool and to introduce Geometer's Sketchpad (GSP) software within a problem-solving context. The problem could possibly be worked with paper and pencil strategies, however, the interactive sketch created a visual aid for the students making the specific probability problem easy to solve The visual representation also helped direct students in working out a generalized solution in symbolic form.

The "Wait for a Date" probability problem and the accompanying interactive sketch can be found at http://mathforum.org/mathtools/tool/17866. It is a free activity that may be downloaded and used in class. See Appendix A for the problem, instructions, and sample pictures.

Below are a few comments students made when evaluating the activity:

- "I really liked this assignment. I also thought that it was a very good assignment to start off the term with because it wasn't very hard for the user. It was a very user-friendly activity and it dealt with a real life application of math. It also required us to think about the graph and how to work with it using algebra. I felt like I learned a lot from this activity about sketchpad and about how to deal with simple real life problems."

- "I think that the visual aid that was provided was great. For me there are only two advantages to technology. They make the problem fast and they are able to illustrate what is going on. In the case of this project it did both."
- "From this assignment, I initially learned how to observe things using GSP. I learned that with problems such as that, having technology is beneficial to solving the problem because we could effortlessly change times and observe many different instances of them meeting so we could easily see when they would and would not meet. I liked being able to use the animate to make a ton of different points on the graph really fast. It was easy to see the pattern of the graph with the different colors used."

Example of a Math Task with "Step by Step" Technology Instructions

These types of problems had two main objectives: first, to help students become familiar with the technology and learn how to use it and second, for the students to be able to complete the math tasks with the assistance of the technology. The following problem was adapted from the TI-84 Plus Cabri Jr. Application Manual (p. 26). Other dynamic geometry software with animation capabilities would work just as well in visually depicting the locus. Once the student had constructed the figure with the technology tool providing a visual representation, the mathematics task really began. See Appendix B for the Locus and Animation Task with instructions.

This problem is rather simple; the technology is used mainly for visualizing the locus as it is drawn. The points could have been constructed by lengthy paper and pencil methods, but the dynamic technology is faster, more accurate, and provides a clear visual representation. In order to "prove" what the locus really is, geometry theorems need to be applied. In essence, this problem gave the students the experience of creating, visualizing and then discovering the conic section definition of a parabola by combining ideas from geometry and algebra. This approach is more convincing and lasting than simply writing the definition on the board and drawing a picture with a few points to illustrate how the parabola graph is formed.

A few student comments for this activity follow:

- "The interesting part of this assignment was that I learned that we can also animate points on a TI-84calculator. It was also interesting that I didn't expect the locus to be parabolic."
- "This was a good activity to learn about the calculator. I didn't know that it could do all of these things so it was impressive, but I never really understood what it meant. I did like the directions we were given for it though because it was really easy to figure out how to create it."
- "I learned about locus and angle bisector. I found that you can animate to see what is going on with lines. This problem was OK it showed me a different tool on the calculator. I didn't like how there was no explanation on locus or the result."
- "I liked this one. I thought it was a very cool application of the TI-84. I know I said earlier that I didn't like using Cabri geometry but I actually thought this one was very good. I felt like I learned a lot in this one about properties of graphs and parabolas. I would make this a keeper."

Example of an Extended "Step by Step" Activity

There is an excellent example of graphically and algebraically exploring derivative functions using GSP in [6]. A similar approach can be used to graphically explore the antiderivative. The antiderivative activity was discussed at the NCTM 2004 Annual Conference and subsequently placed on the Internet. The mathematics activities, questions, and directions for making the antiderivative probe are located at http://www.kcptech.com/sketchpad/scott/nctm2004. In this activity, the construction of the antiderivative probe is actually quite instructive in learning about tangents, slope fields and the antiderivative function. The antiderivative probe powerfully illustrates how slope fields can be created from the derivative as well as dynamically demonstrating the concept of obtaining a family of curves. Due to the restrictions on the length of this paper, the sketchpad instructions, and related questions had to be omitted, but they are available from the author or at the above stated web link.

A few student comments concerning the antiderivative activities:

- “This one was a little long but I have to admit that I really enjoyed it. It tackled a complicated math concept and made it much easier to grasp. Technology really helped in this case to see what an antiderivative of a function really looks like and why it looks the way it does. I got a better understanding of this subject because of this assignment and I also learned more about sketchpad.”
- “I learned how to construct many new things on geometer's sketchpad that assisted with the answers to the assignment. I just thought that the technology we used by tracing it using the probe was very neat.”
- “I really enjoyed doing this activity. It took me a long time doing it. However, I learned how to draw an antiderivative graph, and know how it works.”
- “From this activity I was able learn that the antiderivative of a function can be found on geometers sketchpad apart from applying the calculus. Also that the shape of the graph does not change as the y value of the probe changed up or down. The interesting part of this assignment was that I was able to create the probe all by myself. This was my first time to create something on geometers sketchpad from scratch and It was so much fun. I really like the automatic probing part. From here I was able to recognize that automatic is so much neater compared to the manual one.”

Example of A Math Task With "Minimal Technology Guidance"

Problem: Consider a cubic polynomial function with three distinct real zeros. Where does the tangent line drawn from the graph of the polynomial function at the average of two of the three zeros intersect the graph again? Does this property hold no mater what two zeros you average?

For this problem a CAS was used to simplify the messy algebra and to symbolically solve a third degree polynomial equation. A numerical example, worked in class, was provided to illustrate the mathematical concept. Students were then asked to determine if the concept would generalize for all cubic functions. After a little thought, the students determined that the problem would only make sense if the cubic polynomial had three distinct zeros. The students had already worked with the CAS feature of the TI-89 calculator in previous math problems. So, with a little prompting, the students suggested that it might be an appropriate tool to use in working the generalized problem. See Appendix C for the numerical example and proof using the TI-89.

Student comments:

- “This one was cool once you showed us how to do it. I actually really enjoyed this one. It would've been horrible without the technology so I thought it was a good problem to do with technology. I wouldn't really change this problem except maybe giving us a little more direction before we try to do the problem.”
- “I guess that the only reason that I liked this one is because we ended up doing it in class. I had a really hard time understanding the problem. If it isn't explained well then it seems to be hard for the student to get it, even with continual explanation. It should be kept, but I think that this is one of those that the students can re-vamp to make something better.”
- “I learned about where the three zeros would be. I liked that we did this assignment in class. It was confusing to me before we did it in class because I didn't even know where to start. I think that this assignment should either be done as an in class activity or taken out.”
- “I learned from the Cubic Zeros (TI-89) about tangents and their relationships to graphs. I like the idea behind these questions. I was confused on how to start these problems. I was lost on where to begin, but once you explained what we needed to do, I was able to work out the proof”

Example of Mathematics Tasks Without Technology Guidance

The idea for this problem was found at http://ccl.northwestern.edu/papers/rugby. The problem from the Internet is quoted below.

“In rugby, after a “try” has been scored; the scoring team has the opportunity to gain further points by "kicking a conversion.” [i.e. Kicking the ball into the goal - the segment AB in the figure]. The kick can be taken from anywhere on an imaginary line [line CK in the figure] that is perpendicular to the try line [line CB in the figure] and goes through the point that the try was scored [C in the figure]. Where should the kick be taken from to maximize the chance of a score? (In other words, where should we place the point K; so as to maximize the angle AKB?). "

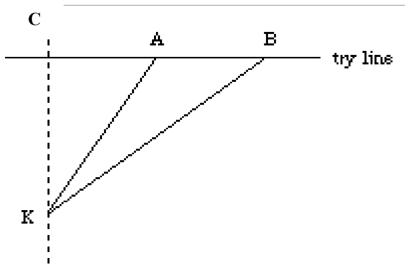

Figure 1

The math assignment given to the students came in two parts. The first part consisted of determining the answer to the above problem for several different “try” places. Then, once sufficient data were collected, the students used regression analysis to determine the best model for predicting the optimal angle. For the second part of the assignment, students were asked to do the original British textbook problem which was to: 1) prove that the optimal place to kick the conversion is at the point of tangency on line CK with a circle containing A and B and 2) prove the locus of points for the optimal angle AKB forms a hyperbola.

Student comments:

- “I learned that at each angle the chances of kicking the ball into the goal increases or decreases. I liked that we didn't have to do this problem by hand like we first started doing. I enjoyed how you brought that aspect of it into the assignment so we thought we were doing it a way easier way. I thought that the proofs were very hard that we had to do at the end.”
- “I thought this was a very clever idea. I used technology very effectively and it was a real world application of math. I felt like I learned a lot in this one about graphs and optimal angles. I was really surprised to find that the optimal angle occurred at the circle tangent to the vertical line.”
- “I really liked this project. The instructions could have been a bit better but that is okay. The message was still communicated. I think that I liked it a lot because it could be approached from so many different ways in order to get the answer. It helped us see more into geometry and trig along with a bit of calculus.”
- “This actually covered some important topics of mathematics as physical application. The activity was fun and at the same time it was really useful. The proving part taught many fundamentals of proofs. The most interesting part of this assignment was the study of angles in the circle. This was a good review of the properties of circles learnt long before. The proving part took a long time and a lot of thinking.”

Conclusion

The student evaluations of the activities were generally positive both in regards to the math assignments and in the use of the technology. All the students felt the course had been beneficial to them and began to recognize how technology can be appropriately used in learning and doing mathematics. Although the students improved in attitude toward technology, some were still concerned about becoming too dependent on technology or that they may become lazy by using technology to do their work. It may be difficult to completely eliminate the "crutch" mentality for some pre-service teachers, even when presented with authentic uses for using technology in working mathematics problems.

One area needing improvement is to help students be able to learn to take a math problem and determine how technology can be used as an appropriate tool in solving the problem. Some of activities were purposely selected because of the good mathematics involved and yet were either ambiguous in explaining the problem or in how to use the technology. These were selected to foster discussion on good pedagogy when working with technology. However, it appears from student comments there needs to be more discussion on how to make a poorly designed math activity better. The students in general were not able to do this. Yet when they begin teaching this will be a critical skill for them to have in order to successfully incorporate technology in teaching of mathematics and for the learning of mathematics by their students. Plans are being made to include more opportunities for students to suggest how technology could be use to work mathematics problems and how to make mathematics tasks even better.

Clearly, one course exposing students to a number of technologies is not sufficient to provide pre-service teachers the knowledge to successfully incorporate technology into the teaching and learning of their students. There must be a means for continued use of technology in future mathematics courses the students take during college along with follow-up professional development opportunities such as workshops and seminars to help refresh knowledge of previously learned technology applications and to expand on new ways to effectively incorporate technology in their mathematics class for the good of their students.

A number of countries currently use *Lesson Study* as a successful way of improving teacher content knowledge and pedagogy. Some school districts in the US have also initiated *Lesson Study* for professional development within schools and districts. *Lesson Study* may prove to be a viable option for assisting teachers in improving their knowledge and skills of math technology as well as developing exemplary technology-based mathematics tasks matched to the curriculum. These well-developed lessons created through lesson study will hopefully use technology appropriately so as to assist students in understanding the concepts better and to enjoy and appreciate mathematics more.

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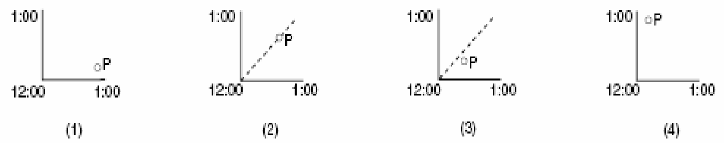
Appendix A

Problem: You and a friend arrange for a lunch date next week between 12:00 and 1:00 in the afternoon. A week later, however, neither of you remembers the exact meeting time. As a result, each of you arrives at a random time between 12:00 and 1:00 and waits exactly 10 minutes for the other person. When 10 minutes have passed, each of you leaves if the other person has not come. What is the probability that the two of you will meet?

Q2 Point *P* is at the intersection of lines through *A* and *B* that are perpendicular to the two axes. The following picture shows four possible locations of *P*. For which two locations does it appear that you and your friend meet?



5. Press the *Do A and B meet?* button. Doing so moves points *A* and *B* to new, random locations along the segment. When *A* and *B* meet, point *P* leaves a green trace on-screen. When the two do not meet, the trace is red.

6. To speed up the process, choose **Display | Show Motion Controller**, and click on the upward-pointing arrow to increase the speed.

To clear the screen, choose Erase Traces from the Display menu.

Q3 Run the simulation for a while. Describe the emerging pattern of red and green points.

Q4 Draw the set of all points *P* for which you and your friend arrive at the exact same time. Describe this set of points.

Q5 Draw the set of all points *P* for which your friend (point *B*) arrives exactly 10 minutes after you (point *A*). Describe this set of points.

Q6 Draw the set of all points *P* for which you arrive exactly 10 minutes after your friend. Describe this set of points.

Use your answers to Q4–Q6 to help with the next two questions.

Q7 What portion of the square formed by the two axes is filled with red points? What portion is filled with green points?

Q8 What is the probability that you and your friend will meet?

EXPLORE MORE

Q9 Drag point *T* to change the waiting time. Now run the simulation again. What is the probability that you and your friend will meet?

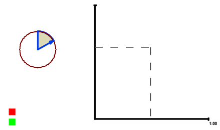
Q10 If you and your friend are willing to wait *t* minutes for each other, what is the probability you'll meet?

Q11 How do things change if you are willing to wait for 15 minutes, but your friend is willing to wait for only 5 minutes?

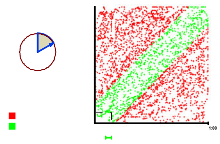
Q12 If you are willing to wait t_1 minutes and your friend is willing to wait t_2 minutes, what is the probability that you will meet?

Partial answers:

What sketch looks like at first



Sketch after many points plotted



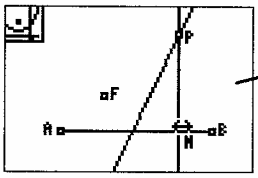
Sketch shows where two will meet or not meet.
It is easy to see the do not meet areas for triangles. The areas can quickly be determined as can the entire area. Then one can find the success area and complete the proble

Appendix B

This task was done as a demonstration using the TI-84 plus SE calculator and viewscreen on an overhead projector with the students following along with their own TI-84 plus SE calculators.

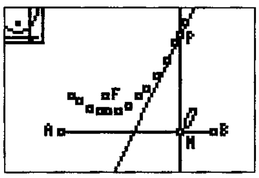
Locus and Animation Example

1. Draw a line segment *AB* and a point *F*.



See Creating Labels for instructions on how to add labels to your drawing.

2. Draw the perpendicular bisector of Point *F* and another point taken on *AB*, named *M*.
3. Draw a perpendicular line to *AB* through *M*.
4. Draw intersection point *P*, of the two lines.
5. Open the F3 menu and select **Locus**.
6. Select point *P* and press [ENTER].
7. Move the pointer to point *M*. The shape of the pointer changes to a double arrow.
8. Press [ENTER]. **What type of curve appears to be displayed?**



9. Press [CLEAR] to quit the Locus tool.
10. Open the F1 menu and select **Animate**.
11. Move the pointer to point *M* and press [ENTER] to animate it. It moves back and forth along the line segment. At the same time, the perpendicular bisector is animated. It stays tangent to the parabola.

Teacher Partial in class helps:

(F2, Segment)
(F2, Point)

(F2, Point then click on AB)
(F3, Perp. Bis.)
(F3, Perp)
(F2, Point then Intersection)

ASK students to guess what they think the shape will be before pressing enter.

Assignment: Prove the locus or set of all points that could be created in the above manner actually do graph a parabola.

Appendix C

Math 308
Exploring cubic functions

Consider the cubic function with roots {-2, 1, 3}. $y(x) = (x+2)(x-1)(x-3) = x^3 - 2x^2 - 5x + 6$

Evaluate the function at the average of any two of the roots, say {-2, 1}. $\frac{-2+1}{2} = \frac{-1}{2}$

$$y\left(\frac{-1}{2}\right) = \left(\frac{-1}{2}\right)^3 - 2\left(\frac{-1}{2}\right)^2 - 5\left(\frac{-1}{2}\right) + 6 = \frac{-1}{8} - \frac{1}{2} + \frac{5}{2} + 6 = \frac{63}{8}$$

So a point on the graph of this function is $\left(\frac{-1}{2}, \frac{63}{8}\right)$.

Now, find the equation of the tangent line to the graph of the cubic function at this point. The slope of any tangent line to the graph would be $y'(x) = 3x^2 - 4x - 5$. Evaluated at $x = \frac{-1}{2}$, we get

$$y'\left(\frac{-1}{2}\right) = 3\left(\frac{-1}{2}\right)^2 - 4\left(\frac{-1}{2}\right) - 5 = \frac{3}{4} + 2 - 5 = \frac{-9}{4}$$

Plugging this information into the point slope form for the equation of a line we get

$$y - \frac{63}{8} = \frac{-9}{4}\left(x + \frac{1}{2}\right)$$
$$y = \frac{-9}{4}x - \frac{9}{8} + \frac{63}{8}$$
$$y = \frac{-9}{4}x + \frac{27}{4}$$

To find the point where the tangent line intersects the cubic function, we set the two functions equal to each other and solve for x.

$$x^3 - 2x^2 - 5x + 6 = \frac{-9}{4}x + \frac{27}{4}$$

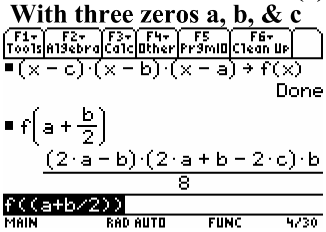
$4x^3 - 8x^2 - 20x + 24 = -9x + 27$	Multiply both sides be 4 to eliminate fractions
$4x^3 - 8x^2 - 11x - 3 = 0$	Set equal to zero
$(x-3)(4x^2 + 4x + 1) = 0$	factor out (x-3) (suspected root)
$(x-3)(2x+1)^2 = 0$	factor completely
$x = 3, \frac{-1}{2}$	zero product property

So the tangent line to the cubic at the average of two of the roots intersects the cubic at the third root. We could show this is true for the other two cases of this specific cubic function. Is it true for all cubics with distinct roots?

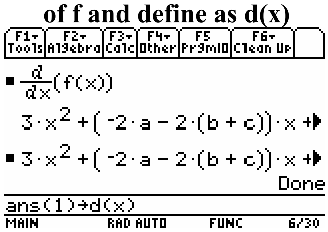
Prove.

Proof Using the TI-89 Calculator

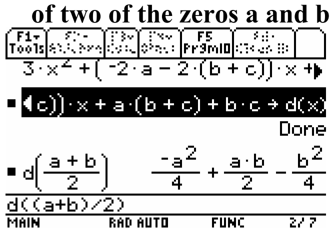
Define a cubic function f(x) With three zeros a, b, & c



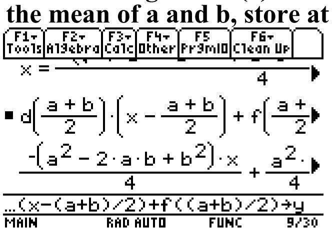
Determine the derivative of f and define as d(x)



Evaluate derivative at mean of of two of the zeros a and b



Find the tangent to f(x) at the the mean of a and b, store at y



Solve the equation f(x) = y to prove the tangent intersect at the third zero