Why and How of Interactive Mathematical Documents on World Wide Web

Ravinder Kumar
rkumar@alcorn.edu
Department of Mathematics & Computer Science
Alcorn State University, USA

Abstract: The real power of mathematics documents comes into play only when they are interactive and reader can communicate with them. Internet related technological advancements have made it possible to create interactive mathematics content for use on the world wide web. Computer algebra systems Mathematica, MATLAB, Maple etc. have proved useful in creating interactive mathematical materials. MathWright has been used to create interactive mathematical modules in Piagetian constructivist spirit. We have created interactive mathematical modules using Scientific Notebook and MathWright. In this paper we trace the transition from a physical document to an interactive document. We consider why interactive mathematics materials should be developed and used, and explain how we create such materials. As examples, we reproduce only the snapshots of interactive components of two of our interactive textual materials because it is not possible to enact interaction on paper medium.

1. A Brief Historical Note on the Evolution of Printing

Process of documentation has gone through various stages in different cultures. In some cultures it started with remembering and passing on from generation to generation, called 'smriti' and 'shruti' in Sanskrit. Engraving on stones, using ink to write on dry leaves, and finally handwriting on paper were some other ways that evolved before the advent of a printing process. It is commonly believed that first printing press was developed by Johannes Guttenberg in Germany in the year 1440 in Europe. However, now it is known and well-documented that first printing process ever was invented in China in the year 868, when Buddhist book "Diamond Sutra" was printed [14]. Chinese used moveable wooden blocks for the purpose. In the year 1041, first ever printing machine was developed in China [11,12]. Since Guttenberg's printing press, printing machines using character blocks evolved through many stages resulting in the only printing method until a few years ago, when computerization and digital imaging took over for a faster and more efficient process. Digital imaging has led to better quality production. All these methods produce 'static' or paper documents and books used in our classrooms, homes, and offices. Most documents produced by a word processing software also fall in this category.

2. Dynamic Document

While static textual documents in the form of physical books and electronic files remain popular methods of textbook reproduction in mathematics, computer technology keeps on empowering us to take further leaps in the presentation of information. In the early nineties, when computer technology had just started spreading its wings, tools were developed using object-oriented programming that produced non-linear and animated documents. This was a definite departure from static documents and ushered in the era of dynamic documents. As an example, ToolBooks multimedia authoring software was used by many, both in academia and corporate world, to convey concepts and ideas with the help of dynamic documents mixed with audio and video (multimedia). Microsoft and Corel introduced PowerPoint and Presentation respectively to bring authoring of dynamic documents to the general masses with reasonable computer literacy. PowerPoint is now a very popular tool for creating effective multimedia documents effortlessly.

Creation of multimedia documents has been state-of-art technique to convey ideas in ways unimaginable earlier.

3. Interactive Document

For general purposes multimedia presentation mentioned in section 2 above is a very effective means of communication. But the needs of scientific community are more demanding. Some of these can be satisfied by the use of animations while others do not compromise with anything less than user-interaction. An interactive document is a computer document from which symbolic, numerical, and graphic tools can be invoked. The results of these computations can be pasted into the document so that each learner has an individual record of his or her explorations. Interactive document takes dynamics of documents to a much higher level. Interactive document responds to user input beyond mouse-clicks. In an interactive mathematical document user can input text strings, numerical values, and mathematical expressions. The feedback consisting of calculations and graphs, alerts etc. is automatically updated according to the user input once an event is triggered by a mouse click or another method. The feedback that a reader gets invokes intense and thoughtful involvement of the reader in contrast to punching a few buttons coupled with passive reading.

Advent of object-oriented programming and artificial intelligence languages has made it possible to create interactive documents that can be used for the purposes of live exploration. A computer algebra system (CAS) is very important for creating an interactive mathematical document. Wolfram, Inc. was first to come up in 1988 with its product Mathematica that allowed high-level interaction in mathematical calculations. Mathematica was followed by Waterloo MapleSoft's Maple and MathWorks' MATLAB. Some other noteworthy computer algebra systems that followed are MATHCAD and DERIVE. Over the years these software have become increasingly user-friendly, resulting in their own communities that are producing more and more interactive materials of diverse nature meeting the needs of masses and specialists alike. As computer algebra systems were evolving, IBM and Mathematics Association of America (MAA) felt a need for developing means to provide a computer-based platform for interactive explorations in mathematics to provide 'experiential learning'. This resulted in the development of MathKit in 1994. MathKit was developed by James White (1946-2004) under IBM funded MAA's Interactive Mathematics Text Project at the University of North Carolina at Chapel Hill. Some of the materials produced with MathKit were showcased, as a preview, in the Joint Mathematics Meeting of January 1994 in Orlando, Florida. James White went on to develop an object-oriented scripting language, called MathWright, using LISP and multimedia tools. He started his MathWright journey in 1994 from Mathkit to MathWright Author to MathWright 2000 culminating into MWAuthor32 in 2001. He integrated Open GL in MWAuthor32 for effectively displaying three dimensional objects interactively. His sudden death in 2004 cut short a process of developing mathematics materials in his 'Piagetian constructivist'[1] style, but not before it had reached a certain level and not before it had made its mark. With the help of projects for MAA, James White trained a community of authors who contributed in the production of microworlds (interactive applets authored with MathWright) that are capable of providing experiential learning. As Internet grew popular James White empowered MathWright microworlds with web capabilities. Sun Microsystems' Java and Microsft's Visual Basic are among other powerful tools for creating interaction, but MathWright tops them all when it comes to mathematics. MathWright library [7] is an online library containing

[1] Constructivism is a learning process in which learners are active participants constructing their own sets of meaning in contrast to traditional approach where their role is restricted to being passive recipients. For Piaget's constructivist approach we refer to Chapman [4].

a large number of Mathwright microworlds out of which about one hundred microworlds were written by James White himself. Among his microworlds is a monumental book on calculus, titled "Calculus in Action". His publications in the Journal of Online Mathematics exemplify how interactive mathematical textual materials may be delivered online.[2]

4. Why Interactive Mathematical Documents/Textbooks?

Educational psychology tells us that there are many learning styles. Learning styles are crucial to motivate students in their learning process. This is all the more applicable in mathematics because of the nature of the subject. NCTM Standards incorporate different learning styles. The Standards require use of technology. Technology makes it possible to use and encourage visualization and exploration in constructivist approach of the Swiss psychologist Jean Piaget (1896-1980). Guided interaction in the beginning and free interaction at later stages can help a learner dive into ordinarily unheralded vistas. Learning capabilities are enhanced by interactive content that answers questions and poses some more by presenting results that may live up to expectation, trigger denial, or throw surprises at the reader. It is like an experience that can be compared to living a life rather than reading a biography. Sure, having a physical book in hand provides ready accessibility, but if experience of many instructors using online materials and the data collected by many academic portals[3] in support of their systems is any indication then interactive documents do have a definite upper edge. They can provide a new learning experience in constructivist spirit whereby students can learn to study and explore mathematics on their own by using hints and instant help that can be built into interaction.

5. Some Issues Involving Use of Interactive Mathematical Texts

An interactive document is a special type of a computer document. Though it can be used on individual computers, Internet is an appropriate and efficient mechanism to deliver it in order to derive maximum benefit.

Availability of Structure

Report on Critical Issue, "Technology: A Catalyst for Teaching and Learning in the Classroom" [1] considers pros and cons of the use of technology in details. In reference to year 2005, the report quotes from an article by Fox, E. [5]:

> "The National Center for Education Statistics reports that there are virtually no differences in Internet access between poor schools and wealthier schools any more, as Internet access has steadily been increasing in public schools over time. Technology being infused into the schools is ongoing, unstoppable, and necessary. Thus, school use and access to new and current technologies is on the rise and more and more states have established technology standards for students, teachers, and administrators. Teachers have begun to use the Internet more frequently as a valuable tool in their instruction:

---

[2] The above narrative of James White and MathWright is based on [2,3,7] and our personal association of four years with him. We have dwelled on the history of the development of MathWright in length because of two reasons: 1. MathWright's promise as an experiential learning platform, and 2. because we use this software for creating our interactive mathematical modules.

[3] We have refrained form giving publishers' data as it is likely to be biased towards their own product when compared to similar products even though the survey is often conducted by well-known universities. Interested readers may visit www.hawkeslearning.com and www.educo-int.com, for example.

Seventy-seven percent of public schools had a majority of teachers who used the Internet for instruction during the 2003–04 school year, up from 54 percent in 1998-99 … with 73 percent of high-poverty schools and 71 percent of high-minority schools having a majority of their teachers using the Internet for instruction."

On the Internet usage and its availability, we would like to quote a prediction by technology experts and scholars in a 2005 survey on "The Future of Internet" conducted by PEW Internet and American Life Project [13].

"57% of them agreed that virtual classes will become more widespread in formal education and that students might at least occasionally be grouped with others who share their interests and skills, rather than by age."

Prohibitive Costs of Books

Educators know that many students cannot afford to buy books. One of the reasons for this is the prohibitive costs of the printed textbooks. The size of mathematics and science books being produced discourages reading by even an above-average student. This leads to a very sketchy and confused learning experience. Texas Board of Education is one of those educational institutions that advocated the development and use of interactive textbooks as early as 1998 under the leadership of Jack Christie. Christie's endeavor has lead to "Technology Immersion Project" (TIP). TIP is implementing an entirely new concept in educational technology—total immersion of teachers and students in technology. Starting 1999, Texas Board of Education has been able to adopt electronic textbook materials in arrangement with publishers and other sources in most of the courses in K through 8; in at least eight courses including mathematics at the high school level all the way up to grade 12 [10]. The information given here is based on the resources provided to us by Ms. Karan Kahan, Director of Technology Applications, Instructional Materials, and Educational Technology, Texas Education Agency [9].

6. Why World Wide Web?

With the advancement in delivering contents through Internet and Internet's growing popularity, world wide web is a very convenient, effective, and efficient way of delivery. It is most suitable for readers to work with interactive documents, and it does so at a fraction of a cost of single-user software cost. It is lot cheaper than physical books. In fact, most publishers now have their own online portals where they have started providing access to interactive testing/tutoring at a nominal cost. Some of them have even started allowing instructors to post supplementary materials on their portals to provide more customized resources to students. Even mathematics departments and professors of many well-known universities are increasingly posting interactive documents on their websites.

7. Production of Interactive Documents

Producing interactive documents is more specialized. For completeness they must consist of suitable static text, mixed with animations besides components that make interaction possible. Authors have to be imaginative and more knowledgeable of the possibilities. As noted earlier, most well-known publishing houses of undergraduate materials have established their own portals which use interactive testing materials along with static and dynamic content for improving student learning. We have used computer-aided instructional techniques as well as blackboard portal for many years. Mathematics department of University of Alabama at Tuscaloosa has created a success story by turning around the falling grades and attrition by using the combination of dynamic text with interactive testing. Online learning is being increasingly used in beginning mathematics and science courses. If success stories are any indication then the experiential learning made possible by interactive documents combined with interactive assessment is bound to help student learning. Interactive textual materials can be produced for many mathematics courses, such as, college algebra, pre-calculus, differential and integral calculus, statistics, computational linear algebra, numerical methods, discrete mathematics, mathematical modeling etc. Besides James White's "Calculus in Action", Mackichan's "Calculus: Understanding its Concepts and Methods" and MathWright based "Interactive Algebra" authored by Samad Mortabit of Metrostate University, Minnesota (www.mathloran.com) are some other interactive books that come to mind. There are many more on world wide web. Wolfram, Inc. is an excellent source of a large number of interactive mathematical documents authored with Mathematica (http://mathworld.wolfram.com/). As mentioned earlier interactive mathematics can be found on many online portals and websites of publishers and universities also. MathWright library is full of interactive modules that can be used to create interactive mathematical textual materials. Over the last six years, we have used MathWright to create over twenty interactive microworlds resulting in about two hundred interactive pages on topics in differential and integral calculus, mathematical modeling, numerical methods, number theory, linear algebra, abstract algebra, and pre-calculus. We have used many of these microworlds with advantage in our classes. Most of these microworlds are available via online MathWright library [7]. Currently, we are developing interactive textual materials using interactive pages from our existing and new microworlds. Difference between our interactive modules and interactive textual materials is that an interactive module is a stand alone interactive component, whereas interactive textual material completely develops a topic with interaction built into it with the help of microworlds.

8. Design of Our Modules

In our modules we use text fields for user input, mathwright scripting language to control and generate interaction, buttons and other methods for triggering an action, special fields called mathedit windows to report feedback based on mathematical calculations, graph windows to display graphs for visualization, textboxes to display non-mathematical feedback such as statements and alerts, and tables to input and/or output a data.

A general design of our modules is as follows:

1. We introduce the concept and give examples, like in a physical document.
2. We incorporate animations of some of the scenarios for the reader to appreciate some of the points being made. Animations lend some dynamics to the document.
3. Some examples are given to lay down foundation for developing the concept and methods.
4. We incorporate interactive component. We use interactive component for experiential learning. Now that the reader has been introduced to a topic and its concepts, we ask the reader to explore the concept using the interactive component for better comprehension.
5. Exercises and examples are generated algorithmically for use with interactive component to reinforce comprehension of concepts and skills. Reader is then asked to try these examples and other exercises, manually as well as using the built-in interactive component.
6. Wherever appropriate reader is asked to make conjectures based on his/her explorations.

7. If necessary, proofs of statements are provided.
8. Finally, the reader is guided to use the built-in interactive component(s) to explore further.

9. Development of the Module and Dissemination

We use the following software in producing our interactive modules.

1. We use Scientific Notebook version 5.5 (MacKichan Software, Inc.) to develop the html pages. Reason for using this version of Scientific Notebook is that it can be used to create animated graphs easily.
2. We create interactive component using MathWright (MWAuthor32). MathWright can convert its interactive microworlds into html pages also.
3. We build help/instructions for the use of interactive component either as a part of the html or as a part of the MathWright applet.
4. Since the interactive document, thus created, is an html document, we do not need the scientific notebook viewer to read the documents. However, the reader must download an ActiveX Control, called mathwrightweb, available from MathWright library [7] to work with the interactive component imbedded in the document
6. Reader has two options: either read the document with all its interaction right on the web or download onto personal computer.

10. Examples

This article is created with a word processor, and is therefore completely incapable of displaying the liveliness of the interactive components. However, as examples, we reproduce screenshots of the interactive components of two of our modules.

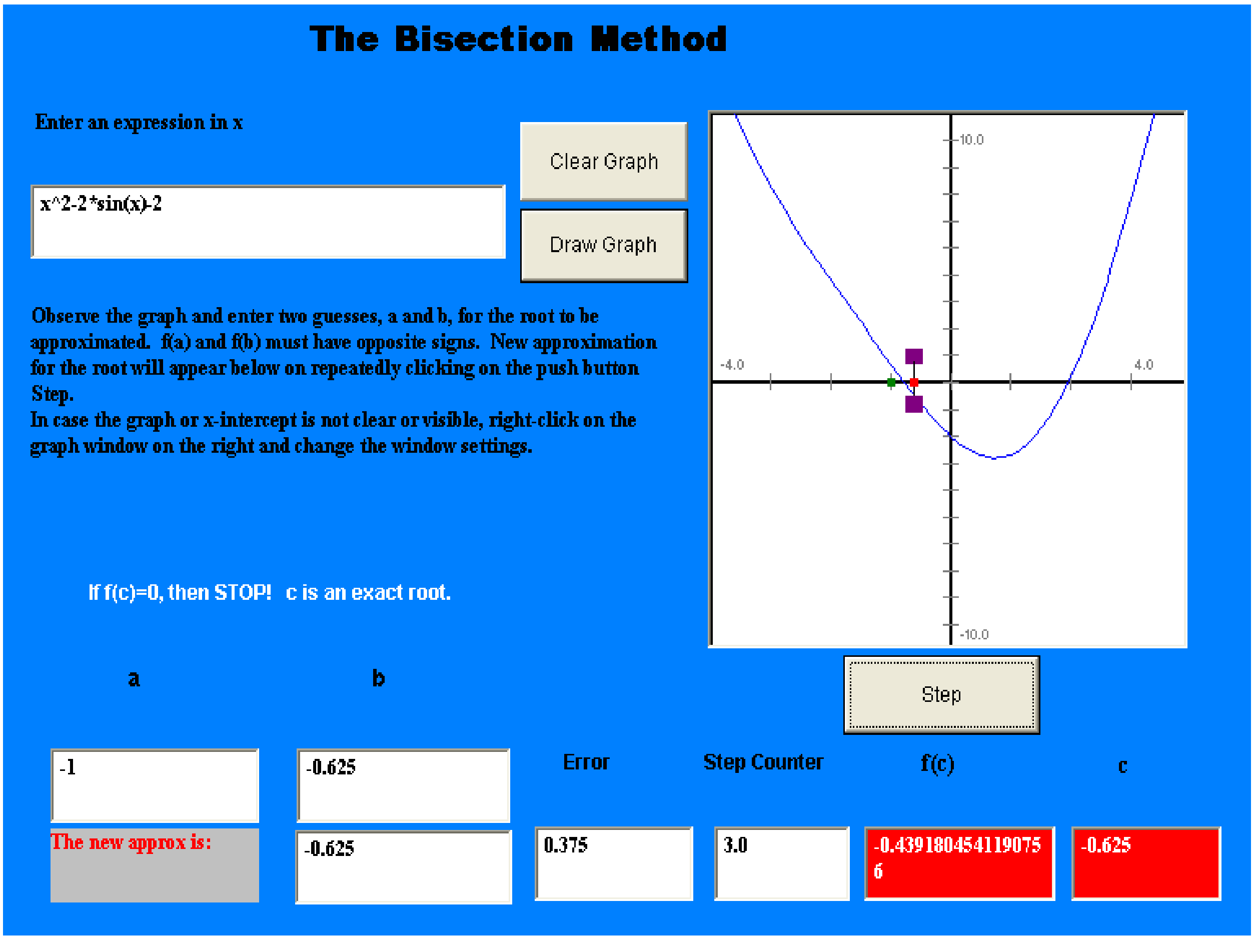
Bisection Method

Bisection method is one of the simplest methods of numerically approximating roots of an equation in one unknown. Given below is screenshot of the interactive component of this module.

Objects of the interactive component are:

1. Input text fields for expression and two guesses
2. Output text fields for new guess, error, step counter, and function value
3. Textboxes for feedback
4. A graph window for visualization
5. Push buttons Draw Graph, Clear Graph, and Step for triggering actions, such as creating graph, clearing graph, and providing feedback with mathematical calculations.

6. Animation objects consisting of blue and red dots indicating two new guesses for the next step, and the purple animation object providing visualization for the new guess at any step.

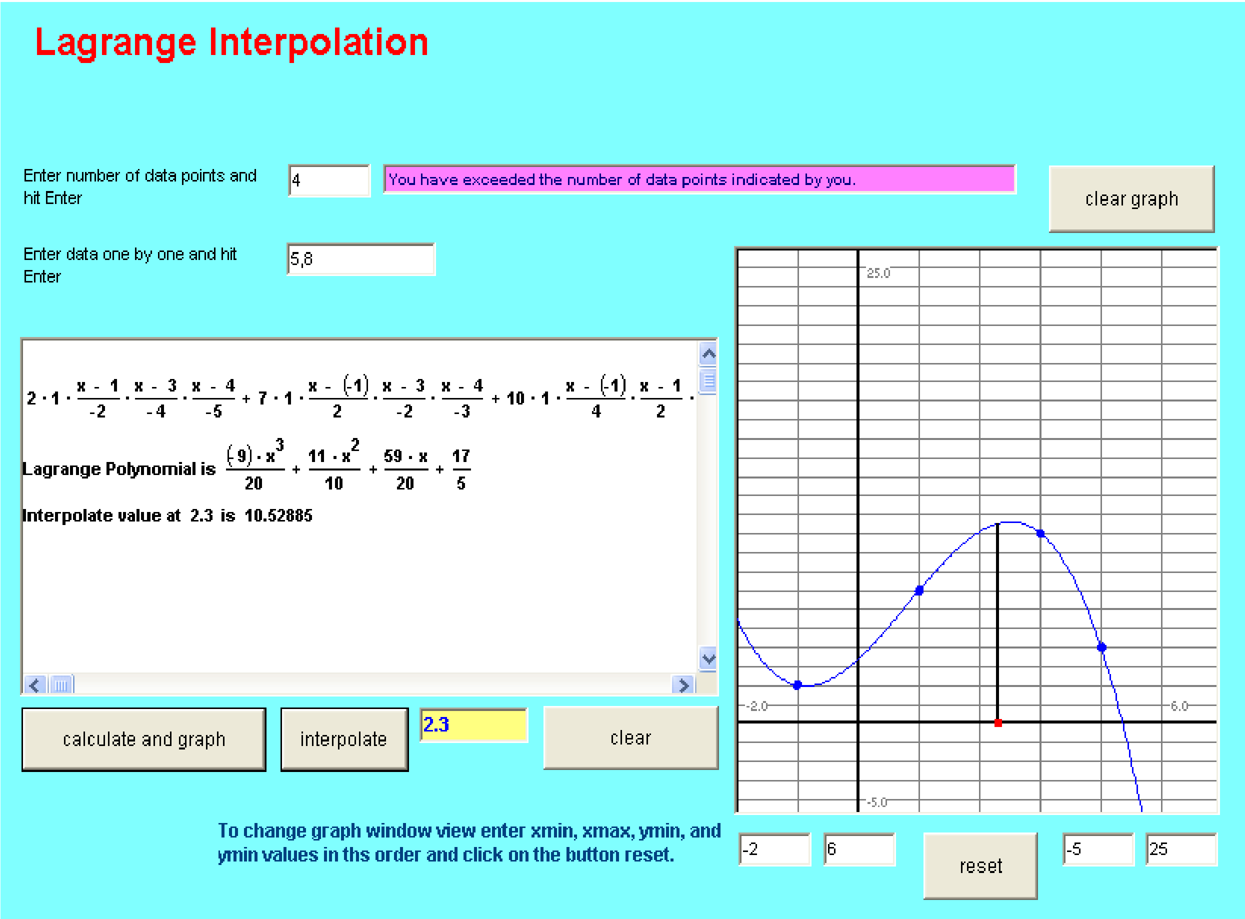


Lagrange Interpolation

Lagrange interpolation is one of the preliminary methods of interpolating data. Given below is screenshot of the interactive component of this module.

Objects of this interactive component are:

1. Input text fields for number of points, data values, *x*- and *y*-range for the graph, and *x*-value at which interpolation is desired. First two input fields trigger action on pressing Enter.
2. A math field for displaying Lagrange polynomial and the interpolated value
3. A graph window for displaying the graph of the Lagrange polynomial
4. Buttons, calculate and graph, interpolate, clear, clear graph, and reset, trigger interactions on click of a mouse.
5. Textboxes giving instructions and displaying alert
6. Animation object consisting of a red dot to indicate interpolation
7. Animated black vertical line, indicating interpolated value



11. Conclusion

With technological innovations in the field of computers and world wide web interactive mathematics materials are available in abundance in the form of Java applets, dynamic html and other means. Interaction puts life into documents to make their message direct and effective by providing instant feedback. By the very nature of interaction in a mathematics document, it helps to explore by giving feedback, thereby enhancing both learning process and learning experience of readers. Putting such textual materials on the web has another advantage for learners because they do not need to carry personal computers everywhere, much less a printed book. We have seen that a physical document such as this one cannot deliver the interaction contained in the two modules we have talked about in section 10, and thus deprives the readers of this article of experiential learning of the two topics. Interactive mathematical texts and documents have a definite edge over physical books. We are not suggesting anything new. It has been proved that an appropriate use of available technologies has the capability of making textual materials in mathematics interactive and such textual materials lead to enhanced learning experience in Piaget's constructivist spirit to provide learners optimum benefit.

References

[1] http://www.ncrel.org/sdrs/areas/issues/methods/technlgy/te600.htm

[2] http://www.mathwright.com/cv.htm

[3] http://www.maa.org/features/101404whiteobit.html

[4] Chapman, M. 1988. Constructive evolution: Origins and Development of Piaget's Thought. Cambridge University Press

[5] Fox, E. 2005. Tracking US Trends. Education Week, 24(35), 40-42

[6] A written document by James White.

[7] http://www.mathwright.com

[8] http://www.tea.state.tx.us/technology/tip/

[9] http://www.tea.state.tx.us/textbooks/adoptprocess/adoptioncycle.pdf

[10] http://www.mackichan.com/

[11] http://en.wikipedia.org/wiki/Printing_press

[12]http://inventors.about.com/library/inventors/blJohannesGutenberg.htm

[13] http://www.pewinternet.org/pdfs/PIP_Future_of_Internet.pdf#search=%22Future%20of%20Internet%22

[14] http://www.bl.uk/onlinegallery/themes/landmarks/diamondsutra.html