# Distance learners and a CAS-based mathematics course: Evaluation and its problems

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## Abstract

The growing use of technology in teaching and learning mathematics has not only changed the way we teach but also what we teach. A mathematics foundation course offered by the Open University (OU) in the UK, based on the computer algebra system Mathcad, reflects both of these types of change. In this paper we present the results of a longitudinal evaluation of this course, giving insight into how distance learners interacted with the mathematical software. In addition, we will explain why the traditional methods of course evaluation need to be extended and revised to become suitable for computer-based courses.

## 1 Background

It is well known that teaching and learning mathematics is undergoing a profound change brought about by the application of computer technology: "Of all influences that shape mathematics [teaching and learning], technology stands out as the one with greatest potential for revolutionary impact" (National Research Council, 1990). Such applications include computer algebra systems (CAS), specialised mathematical software, Web-based mathematical resources and multimedia. Since students and instructors are the main users of these applications, it is reasonable to assume that their experience of such applications significantly influences the success or failure of the applications. A large number of new users are distance learners, one of the most rapidly expanding sectors of higher education. In 1970 the OU was the only tertiary distance learning institution, today there are over 500 similar institutions all over the world. It is therefore imperative to know how distance learners might react to using computer technology in teaching and learning mathematics and to incorporate such knowledge into the design of these systems. This was the purpose of the present evaluation which is, so far as we know, the first of its kind. It represents a continuation of our evaluation of distance learners' reaction to technology in Zand and Crowe (1997), where we reported the evaluation of a distance learning graphic calculator-based course.

The use of CAS in face-to-face teaching has been extensively discussed and at times evaluation reports have appeared in research journals: in Harvey et al (1995) the influence of technology in teaching linear algebra was discussed; Shaw et al (1997) reported on an evaluation of the "effectiveness of using CAS in an Intermediate Algebra Course"; Coutis (undated) reported on the students' reaction to a Mathematica-based course offered at UTS, Sydney. On a different strand, Shaffer (1997) reported how art and design helped a group of students at MIT to learn about symmetry. At a more general level, for example, the monumental work of Noss and Hoyle (1996) has advanced our understanding of the use of technology in teaching and learning matheamtics.

## 2 The present study

In this paper we will report on a longitudinal evaluation, during 1997-2000, of a mathematics course offered by the OU. The course is based on Mathcad and was offered to first year students in 1997 for the first time; each year over 1500 students register for it.
Specific questions addressed in the paper are as follows:

- How do students feel about learning mathematics from a course based on a computer algebra system?
- In what ways can a computer algebra system help students learn mathematics?
- What kind of problems do they encounter?
- Is a large system such as Maple, Mathcad or Mathematica the right software for teaching mathematics to first year university students?

The longitudinal evaluation of the course took place between 1997 and 2000. Our main aim was to see the course from the student's perspective and share their experience. The traditional evaluation method, where feedback is elicited retrospectively by a general purpose questionnaire, was not suitable for our purpose and so we proceeded as follows.

- A random group of 300 was selected from nearly 2000 UK-based students who had enrolled at the start of 1997 and their progress was followed throughout the year. We collected information about their experience four times at regular intervals using specially designed feedback questionnaires (FBQs). The response rate to our FBQs was 48%. We closely monitored the students' progress in interacting with Mathcad by selective telephone interviews.
- During the following two years we supplemented our data by interviewing groups of 25 students each year, monitoring their progress. The students were encouraged to use email for sharing various aspects of their experience of the course and also to discuss their questions about the evaluation.

Technology offered efficient facilities for information collection, such as usage of course material, collecting a list of difficulties encountered each time Mathcad was used, etc. Ryan (2000) contains useful case studies and suggestions for using technology in the context of evaluating faculty members' experience, but we found personal contact to be a productive complement to technology in gaining insight into the experience of students.

## 3 The course

The content of the course was influenced from the outset by how it was going to be taught, i.e. by using Mathcad. It moves away from demanding routine repetitive computations by students (since these are mostly carried out automatically by Mathcad), and instead concentrates more on ideas and concepts. It covers most of the standard topics of calculus such as differentiation and integration. In addition, it contains concepts from computing such as data sets, recursion, algorithms and complexity, plus an introduction to statistics. The central theme bringing various parts of the course together is the modelling of physical phenomena such as growth and decay, velocity, etc.

**Integration of Mathcad into the course**

To give a good idea of how Mathcad is integrated into the course, we will describe a possible study session. The student must have access to a copy of Mathcad and a PC, together with the relevant printed texts (which includes the handbook containing Mathcad exercises). The student can begin by reading the relevant printed text. As the reading unfolds, the student may come across an instruction to move to the computer and use a particular Mathcad file. After opening the file, frames like the one below (Fig 1), for example, appear on the screen.

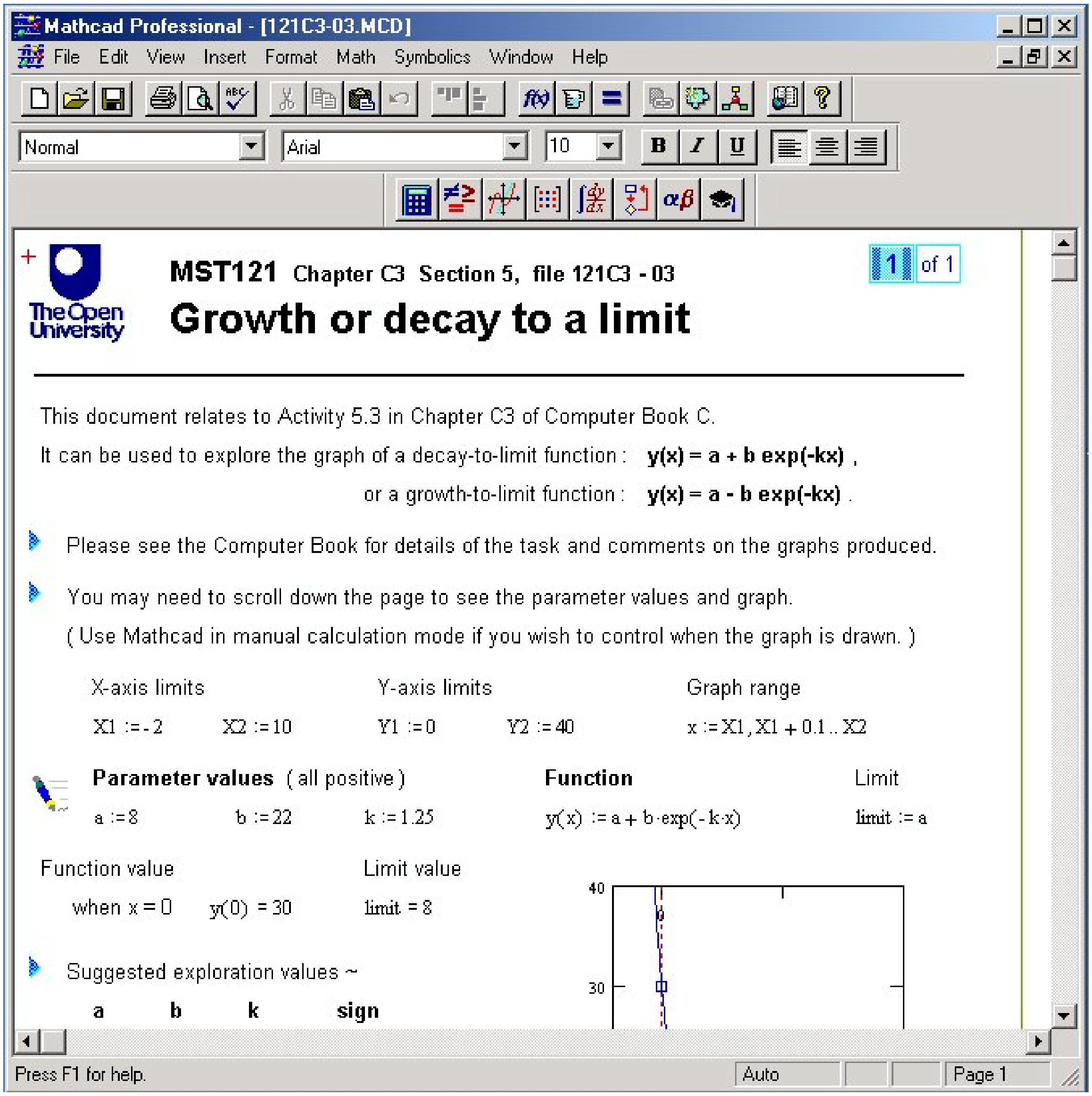


Fig 1: a typical Mathcad file

The student's task from this point on is to read through the material on the screen and complete the suggested activities. These activities can include performing a calculation, exploring the meaning of a concept and visualizing a mathematical object. In the above screen the tasks are contained in the Computer Book, a separate printed text.

What happens if the student encounters a software or a mathematical problem problem?

- Software: In this case he/she can seek assistance from the Students' Help desk at the Open University (a telephone operated system available from 9am-11pm); post a request for help to the FirstClass course conference ; contact his/her local tutor; or use the Mathcad online help.

- Mathematical: Ask the local tutor or seek help through the First Class course conference.

Each student was provided with a copy of Mathcad but they were expected to have their own PCs. The learning materials, which are delivered by mail, included self-instructional printed texts, disks (now CDs) of Mathcad files, plus a handbook describing how to use Mathcad in detail, and mathematical videos supporting certain parts of the course. Students were further supported by tutorials, held approximately monthly in a number of locations throughout the UK, and a telephone help line to help with the computing aspects of the course. The assessment component of the course consisted of both graded homeworks (50%)and a final exam (50%).  The course is the equivalent of a two semester course.

## 4 Main results

1. Charting students progress in learning Mathcad

### 4.1 Rapid progress

A large portion of students had little or no experience of using Mathcad or any other computer algebra system at the start of the course.  However, about two months into the course the situation had substantially changed, with most students feeling confident in using Mathcad at the required level.

Table 1:  Progress in learning Mathcad

| | 1<br>Very competent | 2 | 3<br>Competent | 4 | 5<br>Not at all Competent |
|---|---|---|---|---|---|
| At start of course | 0 | 0 | 1 | 2 | 97 |
| After 8 weeks | 12 | 15 | 34 | 21 | 18 |
| After 12 weeks | 17 | 42 | 26 | 12 | 3 |

### 4.2 Stages of development

Looking at the students' experience one can distinguish three distinct and to some extent overlapping stages in the process of learning to use Mathcad, which we call early, middle and maturity stages. In the early stage, roughly the first six weeks, students were getting used to the alien environment of Mathcad, learning to load and save files, setting up vectors by trial and error, and doing these cautiously and without great confidence. In the middle stage, which roughly occurred during the weeks 7-12, they expanded their ability to use more Mathcad facilities, editing mathematics texts, carrying out computation, graphing etc. They felt much more confident in what they were doing, and this was the defining distinction between the early and middle stages of their development. From about week 10 or so, the majority of students began to use Mathcad for visualising mathematical objects and began to explore and experiment with formulas and concepts at the level required or suggested by the course. The hallmarks of the maturity stage, from about week 12 onwards, were the facility and increased confidence with which they were able to use Mathcad and their skill in using it for computation, visualisation and experimentation with mathematical objects. This, however, did not involve students writing their own programs, or creating or modifying the dynamic and interactive nature of the medium, and so we do not know whether they reached the stage which diSessa (1991) called "deep computational literacy".

As mentioned earlier, one component of the course was a computer handbook which contained detailed instructions about using Mathcad. In addition there was a telephone helpline which students could call to discuss their problems, and large number of students used this facility and found it very helpful; indeed this human support seems to have been critical in all stages, particularly in the early stage. Later in the course some students thought, in retrospect, that they had perhaps been offered too much support and that this did not help them to counter difficulties on their own and learn from experience. These above stages are mapped out in the table below which also contains more details about the progress of the students.

Table 2: Stages in learning Mathcad

| Stage | Main activity | Kind of support they needed | What was acheived | Affective | Developing abilities |
|---|---|---|---|---|---|
| Early, weeks 1-6 | Familiarity with Mathcad environment, Navigation | Basic technical skills to use computer | Getting Mathcad to work: navigating, file processing | Not confident, anxious | Gaining confidence in using Mathcad |
| Middle, weeks 7-12 | Increased know-how about Mathcad's capabilities | In area of using Mathcad for computing, | Editing maths texts, graphing, computing | More confidence, less anxious | Gaining confidence in computing Visualising exploring |
| Maturity, beyond week 12 | Using Mathcad with facility | More advanced questions in all areas | Better knowledge about computing, visualising, exploring | Much more confident User of Mathcad | Much more knowledgable user of Mathcad |

### 4.3 Computer enculturation
We asked the students what they did when they tried to draw the graph of a function or explore, for example, how the "shape" of the graph of a polynomial function changed as the power of x in its formula changed. Below are the responses:

Graphing: Did it with Mathcad 38%
Did it with pencil and paper 62%

Exploring change: With Mathcad 45%
With pencil and paper 55%

These responses indicate that using Mathcad had not become the main tool for carrying out mathematical activities, and pencil and paper retained their traditional dominance. However, a change of attitude towards using the new medium was definitely in progress, which we call 'computer enculturation'. We observed this enculturation developing among all students, even those who came to the course with little or nor experience in using a computer.

## 5 Learning mathematics

In broad terms, students found the course interesting and studied many parts of it without much difficulty. Although we do not intend to discuss their reaction to individual mathematical items in the course, there were three problems which preoccupied students throughout, namely the clash of cultures, completing open-ended exercises and the workload. We will deal with these issues in turn, but first let us see in what ways Mathcad contributed to their learning of mathematics.

### 5.1 Mathcad and the course content
The majority of students felt that Mathcad contributed to their concept learning and did so by empowering them to visualise mathematical objects, experiment with and explore further various definitions and relationships. For example, they found the graphing capability of Mathcad very useful, as it enabled many of them to see what was meant by the limit of a sequence as $n$ goes to infinity. However, a moderate number of students also felt that they were passive observers of the screen rather than participants.

### 5.2 Clash of cultures
By this expression we mean the sometimes traumatic meeting, and at times uncomfortable co-existence, between the innovative content of the course and what is expected by many adult students returning to mathematics after many years' separation. The students, whose average age was about 38, were expecting to see a lot of symbolic and number crunching exercises to be carried out with pencil and paper, a lot of techniques and tricks for manipulating algebraic expressions and a linear presentation of differential and integral calculus. They were disappointed because this course emphasised concept learning and de-emphasised repetitive calculations. If any "bag of tricks" was introduced, it was a set of hints as to how to use Mathcad more effectively, and this was of a kind

different from what they had expected. This disappointment appeared implicitly in comments such as "we are not getting enough practice in doing mathematics" or "the computer is doing everything, all I do is push buttons". This clash is a phenomenon that is likely to be progressively healed over the next few years as the use of the computer as an aid to learning is more deeply accepted and embraced within education. For example, the generation of students who have grown up with the computer revolution feel much more comfortable with it and are more likely to use it in more innovative ways, creating a distinct kind of mathematics and related mathematical software, which will empower them to use the computer as a 'voyage of sense making' (Noss 1995).

**5.3 Open-ended exercises**

The central theme of the course, as mentioned earlier, was modelling physical phenomena such as growth, velocity etc. The course contains a number of open-ended modelling examples and exercises. The majority of students did not encounter any serious problems studying these examples and completing some of the easier exercises. However, when they tried to do other modelling exercises many students, even some of those who had already studied mathematics at A-level and beyond, reported that they found them difficult. "I just don't know where to start" complained a student. One possible reason is that a moderate number of students took the course with insufficient background, for example they had difficulty with basic algebraic operations. Another reason is that - by focusing primarily on using mathematics and de-emphasising abstraction, proof and rigour - students did not develop a thorough understanding of the basic concepts of calculus, which probably did not help them to complete open-ended problems on their own. This is a problem which has affected mathematics syllabuses in many other institutions: Noss (1998) offers a compelling analysis of this problem.

**5.4 Workload**

The students reported that on average they spent 16 hours a week on the course, whereas the range suggested by the faculty was 10-15 hours per week. In the first two months of the course they spent about 25 hours a week because they "spent hours on the computer" exploring the newly found medium, Mathcad. A tutor for the course observed that "A lower profile for Mathcad would have been welcome in early part of the course, in competition between the Mathcad and the *work* it is the work that suffers" (italic added). Here the term "work" presumably refers to learning mathematics. Although we do not accept the implied separation between Mathcad and mathematics, this observation sums up the perceived situation in the first few weeks of the course. However, as the course passed the half-way mark, the study time began to drop and reached the suggested level. In addition to higher than expected study time, students complained that there was "an excessive" amount of printed text to read.

**5.5 Some other issues**

- Final exam results

The use of the computer is sometimes expected to perform miracles and make mathematics into an "easy" subject for almost all students, with a consequent positive effect on their grades. We did not find any evidence for this sudden transformation. Indeed the success rate in the final exam dropped by about 5% compared to the predecessor course (which was not based on a computer algebra system). We attribute this to the workload which, as mentioned above, was fairly high. Had there been Mathcad questions in the exam - and recall that the course was based on it - perhaps the exam results could have been different. However, we have no proof of this.

- Attitude towards mathematics

If change of attitude means a reversal of the culturally induced dislike of mathematics which is common is the West, then we did not find this happening over the life of the course. Taking just one course is not sufficient to create a change of this depth. However, what we did find was that the use of the computer enabled students, for example, to go through a solution to a problem several times until it was properly understood, away from the eyes of others and without the fear of being judged. This was a relief to many students who felt inhibited to ask about the same problem more than once, whether by telephone, email or in a tutorial. This increased the students' confidence in their own ability to learn mathematics, and many students, particularly female students, were pleasantly surprised by it. This, from a psychological point of view, is the start of the recovery from fear and anxiety about mathematics, and this is the beginning of attitude change.

- Mathcad suitability

We can ask whether Mathcad was suitable for the purpose that it was intended to serve. Even in retrospect one can argue either way, but there are important lessons to be leant here. First, CAS such as Mathcad or Mathematica are very large pieces of software capable of performing many computational or graphical operations in diverse areas of mathematics. These large programs were originally written for specialist scientists and engineers to assist them in their research and development work, and not for teaching purposes, although newer generations of the software have added extras for that purpose. So more focused, friendlier and pedagogically oriented software would probably be more suitable than large scale systems such as Mathcad. The good news is that, as we have seen in ATCM conferences, many specialists are actively involved in contributing to this important aim. For example, in ATCM conferences many researchers have reported of the mathematical software that they have developed for teaching purposes in applied mathematics and in pure areas such as group theory and algebraic geometry.

## Summary

In this paper we have reported the basic findings of our longitudinal evaluation of a Mathcad-based foundation course in mathematics offered to first year distance learning students. We reported that students learnt to use Mathcad rapidly. In addition, we reported their feelings about the course, both positive and negative. At the end of the course we asked students whether they would take a similar course again, to our surprise, the overwhelming answer was "yes". We took this high rate of approval as a hopeful sign for the future, and concluded that the course, as the first of a generation of computer-based courses to be offered by the OU, was on the whole successful.

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