Graphic Calculators and Mathematical Modelling in a Program for Preservice Mathematics Teachers

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Abstract

The purpose of this paper is to show the result of a study developed under the program assessment methodology. A basic training program for mathematics teachers is presented, designed and assessed which focuses on the educational use of mathematical modelling and the graphic calculator for the teaching of linear algebra. The study participants were students in their last year of their mathematics degree course at the Faculty of Science at the University of Granada, Spain. Program assessment reported changes in the participants' didactic knowledge which was seen in the design of teaching activities with an algebraic content with the incorporation of the TI-92 plus graphic calculator and the mathematical modelling process.

Introduction

The aim of this research is to assess a training program which incorporates the use of the graphic calculator and modelling, using linear algebra, in the basic training of high school mathematics teachers, and to analyse the didactic knowledge of future teachers arising from the program. For the purpose of curricular analysis, we started from a theoretical structure supported on the following four dimensions: conceptual, cognitive, formative and social (Rico, 1997a, 1997b). This theory also considers that didactic knowledge of mathematical topics must be based on representation systems (Janvier, 1987, Duval, 1995), modelling (Niss, Blum & Huntley, 1991, Houston, Blum, Huntley & Neill, 1997), errors and difficulties (Borassi, 1987), phenomenology (Freudenthal, 1983), the history of mathematics (Fauvel, 1991) and materials and resources. The study was conducted at the University of Granada, Spain, within the training plans currently in force for the academic year 1999-2000.

In the course of this research, we chose to use the TI-92 plus graphic calculator, which is included as a didactic resource in the process of teaching and learning mathematics. The mathematical content involved was linear algebra, as this is conducive to a wealth of important applications in the modelling of real-world situations, as raised by Harel (1998) and Brunner, Coskey & Sheehan (1998), among others. The other hand, the application of technology in algebra is beneficial for students and teachers alike, and one such example is that of calculators which have enabled radical changes to be made in practical mathematics, which sooner or later will influence the development of the school mathematics curriculum (Ruthven, 1997).

The pertinence of the research arises from the basic training program for high school mathematics teachers at the University of Granada, since at present new technologies and mathematical modelling are only dealt with very superficially in this program. In order to conduct the study, a pilot program was designed to incorporate modelling, the graphic calculator and linear algebra in the preparation of teaching activities.

The following questions were asked:
What didactic knowledge do preservice teachers develop by using and incorporating the graphic calculator in school tasks, and how do they incorporate it into their professional knowledge?
What criteria do preservice teachers apply for the use of mathematical modelling and how do they resort to it?
What potentialities does linear algebra offer for the establishment of links and relations between the calculator and mathematical modelling in the teacher's basic training?

In this program, the use of the TI-92 plus graphic calculator is due to its interactive usefulness for multiple representations and its system of symbolic calculus, as well as its growing interest for mathematics teachers and mathematics education researchers such as Kutzler (2000), Ruthven (1997) & Waitts & Demana (1996), among others. But this interest, which is intrinsic for us, is of service to the mathematics teacher's professional knowledge.

The role of the modelling process in sequencing and developing the teaching of mathematical contents leads us to regard it as a key element in basic training. Blum (1991) and Ríos (1995) point out that modelling must be incorporated in the teaching of mathematics. The Cockcroft report (1982), regarding practice in the classroom, maintains that the teaching of mathematics, at all levels, must include "problem solving, including the application of mathematics to everyday situations" (p.71). Also in NCTM (2000) it is put forward that, with the use of technology in teaching, students can "... model and solve complex problems that were heretofore inaccessible to them." (p. 26). These suggestions are considered in the Spanish mathematics programs for secondary pupils which state that the mathematical properties of the different objects of study must be related to the properties of the models obtained, and to interpret the results according to the situations proposed.

The choice of linear algebra is based on the curriculum designed by the Junta de Andalucía's Department of Education (1992). This document establishes that in the nucleus of Algebra for Compulsory Secondary Education (ESO) the solving of linear equations and systems with two equations using different methods must be included. Similarly, applications of algebraic methods must be considered in order to solve mathematical and real-world problems. Within the diversity of situations which can be resolved algebraically, the importance of those whose formulation implies the search for one or more data stand out.

Program expectations are directed towards the didactic knowledge of preservice mathematics teachers, and more specifically we conjecture that the designed program will improve their didactic knowledge by intensively integrating the graphic calculator and modelling processes with linear algebra.

Methodology

The study was conducted with students in the last year of their mathematics degree course, at the Faculty of Science at the University of Granada, specifically the academic year 1999-2000. A group of eight (8) students volunteered to participate in the application of the training program on the graphic calculator and modelling in mathematics teachers' basic training. They followed a course-workshop for 20 days. The program's design is based on two main curriculum organizers: modelling and the graphic calculator as a resource in a mathematical context of linear algebra. This paper is a pilot study, which is set within the framework of the program assessment methodology (Fernández-Ballesteros, 1996, Worthen, Sanders & Fitzpatrick, 1997). The dimensions considered in program assessment are summarized in the following table:

Table 1: Program Assessment Methodology

Questions	**Analysis dimensions**	**Current proposal**
Who to assess?	Group of Program Agents	Preservice teachers Program Teachers
Why assess?	Objectives	Didactic knowledge of modelling and the graphic calculator (GC) in the teaching of linear algebra.
What aspects of the program are assessed?	Program components (from the method, content and organization)	- Graphic calculator (GC) in the teaching of mathematics - Modelling process in the teaching of linear algebra - Linear algebra in the solving of real-world problems - Design of teaching activities with an algebraic content.
What to assess in the preservice teacher?	- Skills - Didactic knowledge	- Instrumental handling of GC - Articulation of the GC and modelling as curriculum organizers.
When to assess?	Work sessions	- Input (preliminary task). Didactic knowledge. - Process (at each session). Didactic knowledge. - Output (last session). Didactic and attitudinal knowledge. General appraisal of course-workshop.

The overall objective of the assessment of this year's program is to provide information for a general program of basic training for mathematics teachers at the University of Granada, in the Didactics of Mathematics.

Assessable behaviours were skill and didactic knowledge. This refers to the instrumental handling of the graphic calculator and its articulation with modelling. Also didactic knowledge, that is to say, the use of these organizers to plan tasks to teach and learn mathematics.

The course-workshop, as it was structured, lasted 18 hours. Furthermore, each preservice teacher devoted extra time to carrying out activities with the graphic calculator outside the work sessions. The stages and moments when assessment is carried out are determined by the start of the course-workshop (input), each of the work sessions, with a view to a process assessment, and the end of the course-workshop (output).

Results

When analysing the performance of participants during the course-workshop, there was clear proof of changes in their teaching abilities with the inclusion of the graphic calculator and mathematical modelling.

Before the course-workshop began, it was proposed that the participants should carry out a preliminary task in order to identify how well they were able to initially use the calculator and modelling. The participants were asked to:
1. Identify a *real world* situation which can be modelled using linear algebraic concepts. Write the situation and the corresponding model.

    What relation is there between the situation and its corresponding model?
    What can be said about the situation from the model?
    Does the situation only allow one model or can it have several models?
    Comment on any interesting aspects of the situation and how the model is used.

2. Detail the steps carried out when designing each model and indicate the difficulties encountered.

By way of example, one of the problem situations devised by a participant is presented below:
"*At the beginning of the year, Juan bought 2 ballpoint pens and 8 felt-tip pens from the stationer's for 1,110 pesetas. His friend Sara bought 3 ballpoint pens and 5 felt-tip pens and they cost 825 pesetas. How much does each article cost?*"

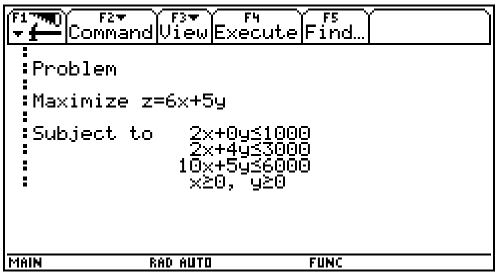
The model proposed for this situation is set out below, where x and y represent the price of a ballpoint pen and a felt-tip pen, respectively:

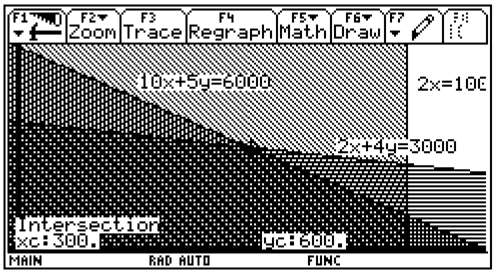
$$\begin{cases} 2x + 8y = 1110 \\ 3x + 5y = 825 \end{cases}$$

When examining the productions devised by the participants, we made sure that the problems situations drawn up were within the understanding of the schoolchildren who the preservice teachers would be teaching. In relation to the questions proposed in the situations, these have a closed structure so that the problems have a single solution with little margin for other criteria and interpretations to be considered which provides a more enriching discussion about the problem situation and the modelling process, respectively. Likewise, we find that when modelling is applied in each situation, the corresponding data interpretation is not performed; that is to say, the process ends with the results obtained in the algebraic calculations.

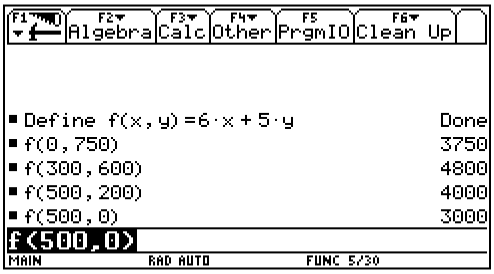
As the programme developed, participants observed how, in each of the activities, the graphic calculator and modelling were incorporated in the activities carried out, noting the use of the graphic calculator for experiments, the definition of mathematical expressions, the use of scales, the solution of problems and use of commands for graphical representation, in the light of the modelling process; this all became apparent in the didactic incorporation of the graphic calculator and modelling in the design of teaching activities. It is important to highlight that some of the difficulties faced by the participants when using the graphic calculator related to the cross between variables and the representation of straight vertical lines.

Below we present one of the productions where the preservice teachers used the didactic usefulness of the calculator when modelling situations related to linear programming. When modelling a situation related to the profits of a company, for example, the following screens were presented:

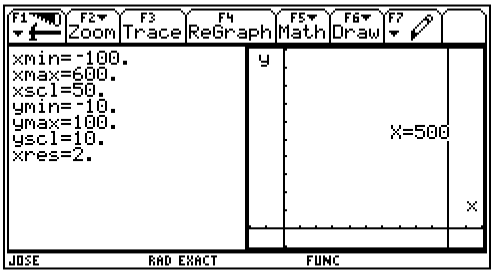




And when evaluating the objective function in the corners of the feasible region, the result was:



In this case, the preservice teachers also resorted to the didactic use of the calculator and recognized the teaching potential of the text editor, and the graphic and symbolic capabilities. However, the participants found it difficult to represent the straight vertical lines on the calculator screen, for example the straight line *x*=500. They managed to solve this by using the limitations which the calculator has regarding its "infinity"; the function defined by $y=10^3(x-500)$ was graphically represented which generated a "straight vertical line" as can be seen on the right side of the following screen. On the left side, the window configuration (window editor) appears:



From the above, it was considered that by taking sufficiently large slopes (infinite for the calculator), graphs with straight vertical lines could be obtained. Moreover, the preservice teachers recognized the importance of knowing the limitations of the calculator (e.g. infinities and infinitesimals) in order to derive teaching benefits from these.

In the last course-workshop session, the following activity was carried out in order to identify those teaching abilities where it would be possible to visualize the incorporation of the graphic calculator and modelling in the teaching of linear algebra:

"Let us suppose that a secondary teacher needs to devise a teaching activity to teach the application of the systems of linear equations. In order to achieve this aim, take the following situation: In a café, the total cost of a small soft drink, a medium drink and a large drink is 2.34 euros. The costs of a large soft drink is 1.75 times the cost of a small one. The difference in price between a medium soft drink and a small one is 0.20 euros. Assuming that teachers know the modelling process and will use the GC together with their students,
a) What questions could be formulated so that it is necessary to use modelling and the GC?
b) How could the sequence of activities to be followed by the teacher be ordered, in order to achieve the objective?
c) Suggest at least two aspects to be evaluated and to carry them out.

Regarding point a), some of the answers were:

Proposal 1: "*Three friends ordered 3 soft drinks and they want to know how much they all have to pay if everything costs 2.34 euros, a large soft drink costs 1.75 times as much as the small one and the difference between the medium one and the small one is 0.20 euros. Then I propose the following questions:*
Resolve the system obtained using the graphic calculator, following the steps given below: 1) Express the equations, 2) Solve using the graphic calculator, and 3) Check the coherence of the results
How much will everything cost if the small soft drink costs 0.65 euros?
Could one solution be that the large soft drink costs 2 euros and the price of the medium is 1 euro?"

Proposal 2: "*How much do 5 small soft drinks, 2 medium and one large cost? How many soft drinks can I buy with 5 euros? The pupil is told that the commands Solve( ), Matrix and rref( ) can be used*".

In general, in the questions provided by the preservice teachers in answer to point a), it could be seen that open questions had been included, which could create an atmosphere of search and investigation for secondary pupils; for example, while they consider that unitary prices are an aspect to be determined they also believe that these could lead to other questions which would enhance discussion. This might also indicate that the participants have applied the modelling process as proposed in the course-workshop, that is to say as a dynamic process which presents challenges for the person who applies it to certain real world situations. There was no evidence of this feature in the preliminary task, which indicates changes in the participants' teaching capabilities regarding the modelling process and its respective application.

Likewise, it is important to highlight that in intermediary sessions, the preservice teachers expressed their opinions about the difficulty of using this type of question. Nevertheless, some of the approaches proposed by the participants clearly indicated the use of the graphic calculator, which demonstrates an attempt to include the graphic calculator and modelling in teaching.

In connection with point b), there were proposals such as:

Proposal 1:
"i*) Create the problem. Discussion, ii) Consider it, iii) Discover the known factors, iv) Teacher's questions to the students: How would you do it?. Suggest questions and v) Results, comments and assessment*".
Proposal 2:
"*i) Determine the unknown factors and consider the system, ii) Weigh it up, iii) Solve the problem using various methods with the calculator, for example in graphic and algebraic form, and iv) Check the solutions and interpret them*".

In their entirety, the sequences of activities shown reflected: 1. Non-traditional tackling of the problem, attaching importance to motivation, approach and discussion of the problem situation, 2. Participation of pupils using a heuristic strategy to propose questions to them, which contributes to the simplification and abstraction stages in the modelling, 3. They suggested that the graphic calculator be used in the modelling stages, but did not specify how important its use was, 4. Search for the understanding of the proposed situations using the formulation of related criteria and the interpretation of results, and 5. Interest as a result of the pupils exploring other avenues and finding different ways to solve the problem situations.

Regarding point c), i.e. evaluation, some of the proposals were:

Proposal 1:
"*i) Not only the results but also the procedure would be evaluated if the pupil carried out the steps correctly, ii) Evaluate the alternative ways of doing the exercises and iii) From the graph drawn on the calculator ask questions about it*".

Proposal 2:.
"*i) I would give them a similar situation and I would assess how they solved the problem, ii) I would also evaluate their capacity to understand and master the problem, making similar questions to those in section a), and iii) I would let the pupils solve the problem in another way which they consider suitable for solving the given situation.*

The proposals relating to evaluation are quite general, although they heightened aspects such as: the use of modelling in evaluation, assessment of alternative ways of carrying out the problems, evaluation of the process rather than results and evaluating the productions relating to the problem situation without taking into account how well the calculator is used in the same way as there is no assessment of how well pupils use paper or pencil.

All of this enables us to deduce that the preservice teachers demonstrated teaching capabilities by systematically applying the stages which the modelling process involved with the support of the graphic calculator in the mathematical context considered in the course-workshop, which is consistent with that observed by researchers in the analysis of their productions and in the work carried out in each session. Some of the teaching capabilities identified were: use of different systems of representation, use of trial and error as an experimental means to formulate and control situations around mathematical models, use of the GC to find models from real data in keeping with secondary education, use of the CG's text editor and double screen for teaching purposes, detailed explanation of the concepts and procedures in the formulation and solution of problem when modelling is applied.

Programme dimensions subject to assessment

Agents

The programme participants systematically carried out the planned activities. Their individual and group productions were presented at the scheduled times. These productions were the base material for the preparation of this study.

Objectives of programme evaluation

The objective of evaluating the programme focussed on detecting the effects of its application in the didactic use of the GC and modelling in the teaching of linear algebra. From the analysis of the preservice teachers' productions, favourable changes could be recognized in the design of teaching activities which incorporated the GC and modelling.

Programme components

In general, acceptable incorporation of the four programme components was achieved (the graphic calculator, modelling, linear algebra and the design of teaching activities with an algebraic content); likewise, the need to replant the programme was identified with additional hours for its application, since the need to dedicate more time to participants' interaction with the GC was recognized.

Skills

The application of the programme helped participants to recognize the importance and to become more proficient in the use of modelling with the support of the GC when teaching linear algebra. The skills attained became apparent as the programme progressed, and in particular when the preliminary task was compared with the final task. In the latter task, it was clear to see the extent to which the programme contents had been understood. In general, it was noted that important didactic capabilities had been gained for the professional teaching of mathematics, such as the importance of motivation in the classroom, approach and discussion of problem situations, together with the incorporation of the graphic calculator and modelling in didactic activities.

Conclusions

The application of the programme favoured the incorporation of new didactic capabilities to include the graphic calculator and modelling when teaching and learning linear algebra and connections to be made between mathematical concepts and real life, as suggested by Edwards and Chelst (1999). Likewise, the preservice teachers recognized the importance of the speed of the calculator as an aid in the classroom to carry out a greater number and variety of problems, enabling the teacher to dedicate more time to significant teaching so as not to continue devoting time to obsolete skills, as Waits & Demana (1996) suggested. The participants' productions related to: the systematic application of modelling in solving real world problems; the use of experiments to solve problems; the use of the graphic calculator in abstraction and solution phases corresponding to the modelling process; the use of the graphic calculator's potentials (text editor, double screen, Cabri Géomètre programme, among others) for didactic purposes.

Regarding modelling, preservice teachers proposed open problems with the purpose of developing pupils' intellectual autonomy, coinciding with Hodgson's proposals (1997) referring to the advantages of using open situations to develop competence in problem-solving. The application of modelling in the preservice teachers' productions provided evidence of the interest in highlighting the active role of pupils in their learning processes and the positive consequences for problem-solving. The approaches given to the different problem situations were aimed at developing pupils' inventiveness and understanding of algebraic concepts through modelling, as well as to awaken interest in new situations and to obtain a more comprehensive vision of mathematics. These considerations are consistent with Socas, Camacho & Hernández (1989), Blum (1991), Stewart & Pountney (1995) and Rios (1995).

Regarding the importance of linear algebra for teaching, the preservice teachers suggested contexts which would allow algebraic concepts to be used for the application of modelling and the recognition of different ways of teaching.

In the evaluation of the programme application, favourable didactic changes were detected in the preservice teachers which could be seen in their productions. This means that the methodology used for the development of the programme helped to create a dynamic and reflective atmosphere; similarly, the programme contents were able to be adapted to the training needs of the participants. However, there was a clear need to incorporate activities which would help to reflect on assessment in mathematics using modelling and the graphic calculator.

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