Allowing the Uses of the Computer in a Mathematical Competition

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Introduction

In the "Regulations Pertaining to the International Mathematical Olympiad" it is clearly stated that "books, papers, tables, calculators and computers may not be brought into the examination room." Let us imagine that a different sort of mathematical competition were to be held in which students are allowed to use computer. We are to address issues, problems and new opportunities arising from such a tournament. Let us call this hypothetical event the Computer-Mathematics Competition (CMC).

Basic Philosophy

The format of CMC should model after the International Mathematical Olympiad (IMO). The goal of CMC is to promote the uses of computer as a tool both for problem-solving and for mathematical discovery. Through this competition talented young people in all countries are encouraged and challenged with problems in mathematics whose solutions require the combined effort of human reasoning and the machine power. The meeting will create opportunities for the exchange of information on how the computer activities may be integrated into mathematics curriculums. Just like the sports competition, the event aims to foster friendly relations between the enthusiasts throughout the world. In this competition strong emphasis is placed on the cooperation between the individual human mind and the machine. The result of the competition serves as a feedback to the software industry on the future improvement of the product. The competition will also be valuable to mathematics educators in charting the future course.

How is CMC Different from the International Olympiad in Informatics?

Just as the Computer Science is a subject different from Mathematics, the proposed CMC should be regarded as a mathematics competition, not as a competition in informatics. CMC poses mathematics questions exclusively. There exists a well-established informatics competition, the International Olympiad in Informatics (IOI) [4]. The sort of questions raised in IOI are as ingenious as the ones

appearing in mathematics competitions. Solving IOI problems also require the combined effort of human reasoning and the machine power. Questions given in IOI can serve as an important guideline for CMC. It is, however, impossible to place the IOI problems under the same category as the ones occurred in most school mathematics curriculum. CMC ought to resemble IMO more than IOI.

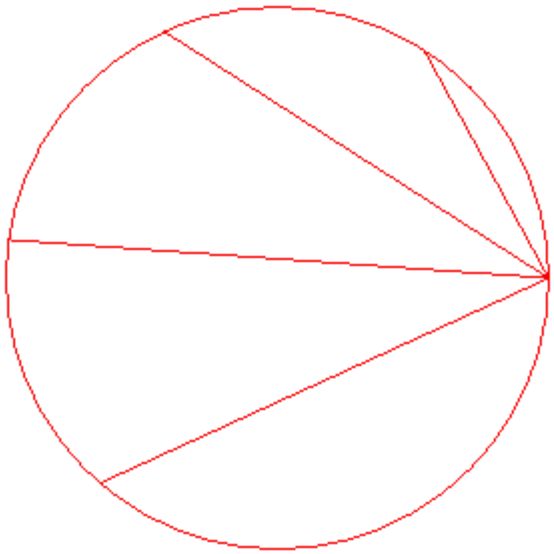
Subject Areas

The main subject areas of the IMO consist of Euclidean geometry, number theory, combinatorics, inequality, functional equations, algebra and recurrence relations. Each area has a large pool of problems dating back to the dawn of the civilization. With the advances of the new breed of mathematics computer systems, many of the classical problems are readily convertible to computer-mathematics competition questions. Like the process of designing problems for IMO, it requires human dedication, talent and experience to craft problems solvable with the aid of the computer. The content and the level of difficulty of each individual CMC problem depends on the type of software allowed. Here are some possible examination questions:

- Problem in Trigonometry: Express tan(nx) as a function of tan(x).
- Problem in Algebra: Are there infinitely many trinomials of the form

  $$x^n + ax + 1, \text{ (a nonzero)}$$

  that can be factored as a product of two lower-degree polynomials with integral coefficients?
- Problem in Function Theory: Find a polynomial in two variables that enumerates the integers.
- Problem in matrix: Let A be a two-by-two matrix. Express the trace of A^n as a function of the trace t and the determinant m of A.
- Problem in geometry: Find an algebraic invariant suggested by this animation:



Level of Difficulty

The difficulty of each individual problem should be ranked by at least two criteria: How hard is to solve it without the assistance of the computer? How hard is to solve it with the assistance of the computer? Among the various software environments, we should ask which ones make the problem easier? Comparison among the various software environment serves a useful benchmark for the mathematics software industry.

Tools and Libraries Allowed

The ability of each computer program may be extended by expanding the amount of libraries to be included. The debate centered around this topic may be long and interesting. Do we only allow the most general library be used? If special-purpose libraries are allowed, are there restrictions imposed on what level of specialty be included? Do we allow the full access to the Internet? Do we allow help library that student may use as memory bank of mathematics knowledge?

Static vs. Dynamic

Each year we see interesting "new" problems appearing in IMO. By and large one may conclude that these are the favorite trends of the year. The progress of mathematics has been a long, tedious process. The duration between the inception of an idea till the formation into an IMO problem takes such a long period that it appears almost static. On the other hand, the progress in computer software and

hardware has been relatively fast. The same mathematical software that made people jumping ups and downs years ago are still making people jumping ups and downs, but for some totally different reason. Therefore, it is possible that the style and contents of CMC problems changes according to the development of the technology. People will view CMC as a more dynamic competition.

Organization

With IMO as the model, the format of CMC should not be too different. The main governing body of IMO is the Advisory Board. It is the hosting country that handles work directly related to the competition. The degree of complexity of the hosting country preparing for IMO appears to be in direct proportion to the number of participating teams. When IMO was held in Canada in 1995, the committees formed were: Board of Directors, Administrative Officials, Academic Committee, Communications Committee, Finance Committee, Fund Raising Committee, Problems Selection Committee, Local Arrangement Committee, Public Relations and Events Management Agency [1]. Problem Selection Committee has always been at the center stage of the organization. It is this group of people who make up the academic arrangements. The task of reviewing and selecting of problems is the most important undertaking. On the practical side, the problem of finances and fund raising cannot be discounted. In early 1990's there was a mathematics competition held in Moscow in which the uses of the computer were allowed. The event discontinued due to lack financial support. [2]

Impact

In ancient India, mathematical computations were performed in public in the form of contest. The Hungarian Eotvos Competition begun in 1894. The first IMO was held in 1959. China first took part in IMO in 1986. Within a short period the "Olympiad School for Kids" flourished. Contest-training publications are stocked in every bookstore. As technology become more and more widespread, the time is ripe for CMC to make a similar impact.

Conclusion

These are pure speculations. Don't we want to turn CMC into a reality?

References

1. Report on the 36th International Mathematical Olympiad, Canadian Mathematical Society, 1995.

2. A.M. Slinko, USSR Mathematical Olympiads 1989-1992, Australian

Mathematics Trust, 1997.

3. http://olympiads.win.tue.nl/imo/imoregul.html
4. http://olympiads.win.tue.nl/ioi/ioi97/contest/ioi97d1.html

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