Internet course material supporting mathematics programmes offered at RMIT

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Abstract

Two of the on-line mathematics subjects offered at RMIT using the internet are discussed: an introductory mathematical subject that supports a particular Manufacturing Engineering course—with an enrolment of approximately twenty students; and a higher level subject taken by the majority of students studying either Engineering or Applied Science at RMIT—with an enrolment of approximately six hundred students. In the first, web pages deliver the content of the subject using Adobe's acrobat format. The subject is based on a prescribed textbook and is presented using various activities including interactive explorations, discussions and Maple-based exercises. In the second, a complete set of course notes that are integrated with sets of illustrative Maple exercises is provided; this on-line material supports traditional lectures offered in a large-class setting.

1 Introduction

In this presentation two on-line mathematics subjects offered at RMIT using the internet will be discussed: an introductory subject that supports a particular Manufacturing Engineering course—with an enrolment of approx-

imately twenty students[1]; and a higher level subject (covering basic topics that include Laplace transforms, Fourier series and separation-of-variable techniques applied to partial differential equations) taken by the majority of students studying either Engineering or Applied Science at RMIT—with an enrolment of approximately six hundred students. The small enrolment numbers in the first subject permit the use of resource-intensive interactive client and server software; the larger enrolment in the second precludes the use of much of this interactive software and is, from the students' point of view, delivered in a much more passive mode. Delivery of supporting mathematics documentation is via the Adobe acrobat pdf format, whilst live mathematical demonstrations, and student submission of assignments, make use of Maple.

2   The Small-Enrolment Subject

This subject is being developed to support a specific niche market identified by the Manufacturing Engineering department at RMIT. The target student group consists of mature age students, most of whom are already in the work force, who are unable to attend lectures on campus regularly. It is expected to run for the first time in 1999.

The subject is based on a prescribed text [1] that was selected by staff in the mathematics department and a copy of which will be provided, on loan, to each student who enrols in the subject. It is expected that the small enrolment numbers will allow this subject to be presented using various resource-intensive activities: interactive student explorations and discussions facilitated using internet tools available in NetMeeting and FirstClass—these items of software are described below; independent problem solving, supported by Maple; group problem solving built around selected text-book applications, with the concomitant discussions utilising web utilities; and web-delivered Maple-based exercises. (A copy of the most recent student version of Maple will either be loaned to each student enrolled in the subject or available for individual purchase.) These activities are based on material contained within the text, although, mainly for test purposes, other resources are currently being developed.

The explorations and discussions are aimed at generating internet-based interactions between otherwise isolated students which is hoped will assist their understanding of the concepts introduced in the text. These interactions can be either asynchronous (using e-mail and First Class conferences) or syn-

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[1]Examples of the material available can be accessed using the main introductory page which is located at the URL: `www.ma.rmit.edu.au/kepler/manops/MA058.pdf`.

chronous (using the telephone, chat within First Class or more comprehensive communication through NetMeeting). The lecturer's help will be readily available; for example, some of the designated activities will consist of on-line tutorials.

Group problem solving activities—each problem to be group-unique with each group consisting of two to three students—will contribute 50% to the total assessment. Such activities are completed by submitting a Maple worksheet to the designated First Class conference. For assessment purposes, each group submission is required to include a clear indication of the contribution made by each member of the group. The remaining assessment for this subject is based on a final examination.

2.1 Internet software

Microsoft NetMeeting is a very powerful Net conferencing tool and is suitable for applications where synchronous communication is crucial; at the moment, there is only a Windows client. It offers standard collaboration tools such as audio, chat, whiteboard and file exchange.

> **Audio conferencing** uses microphones for communication. Bandwidth limitations relating to time delays can cause difficulty with this method of communication.
>
> **Chat** uses a text-based chat window to provide communication between participants. Chat files can be logged for later reference.
>
> **The Whiteboard Window** provides an extensive interactive whiteboard window. Participants can type in this window, open image files and use a variety of tools, including a highlighter and remote pointer. The whiteboard window is synchronised so that all participants can both see the same image and draw on the same space simultaneously.
>
> **File transfer** allows files to be sent and received on call. Such transfers can be person-to-person or to everyone involved in the meeting.

Additionally, Microsoft NetMeeting allows the user to send and receive video clips, share applications and create links on the user's Web page.

First Class is an easy-to-use communication system with both Windows and Macintosh clients. FirstClass supports electronic mail, news groups, file transfer, electronic conferencing and on-line chats. In this subject,

this software will be introduced to the students first. Most of the anticipated student-student and student-staff-student interactions—whether individually or at the group level—can be satisfactorily handled by this software.

2.2   Maple section

This subject makes extensive use of the symbolic manipulation package Maple. This provides the opportunity for students to explore the problem-solving aspects of the mathematics first introduced through the prescribed textbook. The graded exercises from the textbook are sufficiently simple for the students to be able to do by hand, but for this *introductory* mathematics, more complicated calculations can be left to the package to perform. In particular, students will find Maple an indispensable tool to both perform and submit the project work that forms part of the assessment for this subject.

An important feature of the way that this technology has been incorporated into the on-line subject is that the Maple sessions are not isolated from the rest of the mathematics programme but are fully integrated into it. For each major topic there is an associated Maple activity in which students explore concepts in greater detail by first downloading an electronic worksheet which they can run using their local copy of Maple. The Maple worksheets have the following format: Aim; Mathematical Background; Worked Examples; Independent Problem Solving; and Maple Command Syntax.

The Mathematical Background section summarizes material for the topic most recently covered in the textbook. The Maple commands in the Worked Examples section can be executed by the students and the output (which often takes the form of a graph or an animation) can then be analysed. The examples are fully discussed in text regions in the worksheets. Students are then asked to solve problems themselves using Maple. Often these will be problems from the textbook and some may have been previously completed by hand. A reduced version of the Maple worksheet for Topic 5–Linear Programming is given as an example in Appendix A. (Only two of the three worked examples are reproduced and the section on Maple syntax is omitted. Hyperlinks to Maple help pages and to Maple worksheets for other topics that appear in the actual worksheets are not shown here.)

The Maple activities give students a second 'bite' at important topics and, as has been documented elsewhere [2], their learning is reinforced through having to revisit ideas and apply techniques in different contexts. Other pedagogical advantages of using Maple in the mathematics curriculum include the ability of students to tackle 'real-world' problems and to visualize solu-

tions to problems by using the wide range of graphics and animations built into the package.

3 The Large-Enrolment Subject

This subject has been offered at RMIT for many years. Previously, supporting material has been provided—as part of the department's Lecture Note Series—in the form of printed notes and Maple laboratories. In 1998, this material was modified to be delivered using web technology. Specifically, the lecture notes and Maple laboratory sessions were reformatted into smaller 'bite-size' sections, with each section delivered as printable, coloured, screen-size, Adobe acrobat images. The decision to deliver the Maple laboratories as static Adobe acrobat pages was purely one of convenience. The LaTeX 2_ε code was already available and so the .pdf files could be quickly created. In future, based partly on experience gained in other subjects, it is anticipated that these will also be delivered as 'live' Maple sessions; a clear advantage is the opportunity for student experimentation by, for example, changing parameters in differential equations and replotting the solutions; a clear disadvantage is lack of student accessibility to copies of Maple.

Maple assignments are delivered to the student as 'live' Maple worksheet templates, using web technology, and are only made available for a specified period of time. These describe, in text-based regions, the activities to be performed and are made (uniquely) student-specific using algorithms based on their student number. Students submit this filled-out template for assessment, either in printed form, or as an electronic worksheet—on floppy disc or as an attachment to an email message. The latter method of submission has the advantage from the point of view of allowing automated machine marking, but, for large-enrolment subjects, imposes overheads on local electronic mailing systems.

Appendix—Example Maple Worksheet

A.1 Aim

In this Maple activity, you will find optimal solutions of linear programming problems by first obtaining graphical solutions of systems of linear inequalities. This will involve use of the Maple commands **inequal**, **solve**, **subs** and **with**.

A.2 Background

A linear programming problem involving two decision variables x_1 and x_2 consists of finding the optimal value of a linear objective function given by

$$z = c_1 x_1 + c_2 x_2$$

where c_1 and c_2 are constants and the decision variables x_1 and x_2 must satisfy specified problem constraints as well as the non-negative constraints $x_1 \geq 0$ and $x_2 \geq 0$. The optimal value may be a maximum or a minimum value.

As was discussed in Topic 4–Linear Inequalities, the feasible region of a system of linear inequalities consists of all points that satisfy all of the inequalities simultaneously. In a linear programming problem, the feasible region therefore consists of all points (x_1, x_2) that satisfy both the problem constraints and the non-negative constraints. The feasible region may be empty; that is, there may be no points that satisfy all of the constraints. A point (x_1, x_2) in the feasible region at which the objective function z has its optimal value is called an optimal solution of the linear programming problem.

You have already solved several problems from the textbook on this topic 'by hand'. In this activity, you will learn how to use Maple as an alternative method for solving some of these problems.

A.3 Worked Examples

The feasible region for a linear programming problem may be determined graphically by using Maple*s **inequal** command. This command has been discussed in detail in Topic 4–Linear Inequalities. Before attempting to use the **inequal** command, you should read in the **plots** package first, as follows.

```
>   with(plots):
```

Example 1

Maximize the objective function P given by

$$P = 50\,x_1 + 80\,x_2$$

subject to the problem constraints $x_1 + 2\,x_2 \leq 32$ and $3\,x_1 + 4\,x_2 \leq 84$ and the non- negative constraints $x_1 \geq 0$ and $x_2 \geq 0$.

This problem arose in connection with production scheduling in tent manufacture and was solved by hand in Example 1 on Page 274 of the textbook. The decision variables x_1 and x_2 represented the numbers of two different kinds of tents manufactured each day and the objective function P represented the daily profit in dollars. An optimal solution (x_1, x_2) is one that gives the maximum profit subject to the constraints.

As well as plotting the feasible region, we will plot several *constant-profit lines* (or *isoprofit lines*) obtained by giving P specific values in the equation

$$P = 50\,x_1 + 80\,x_2.$$

This is done by first assigning the expression $50\,x_1 + 80\,x_2$ to a Maple variable P.

```
>   P:=50*x[1]+80*x[2];
```
$$P := 50\,x_1 + 80\,x_2$$

(Note that the subscripted variables x_1 and x_2 are entered as x[1] and x[2] in the Maple command above.) We then obtain isoprofit lines by including equations of the form $P = c$, where c is a constant, along with the system of inequalities in the first argument of the **inequal** command. Here we plot the isoprofit lines for which P has the values 400, 800, 1200, 1600 and 2000. The colour and thickness of these lines (as well as of the included boundaries of the feasible region) are specified in the *optionsclosed* argument of the **inequal** command.

```
>   inequal({x[1]+2*x[2]<=32,3*x[1]+4*x[2]<=84,x[1]>=0,x[2]>=0,
P=400,P=800,P=1200,P=1600,P=2000},x[1]=0..35,x[2]=0..25,option
sfeasible=(color=grey),optionsexcluded=(color=white),optionscl
osed=(color=black,thickness=1));
```
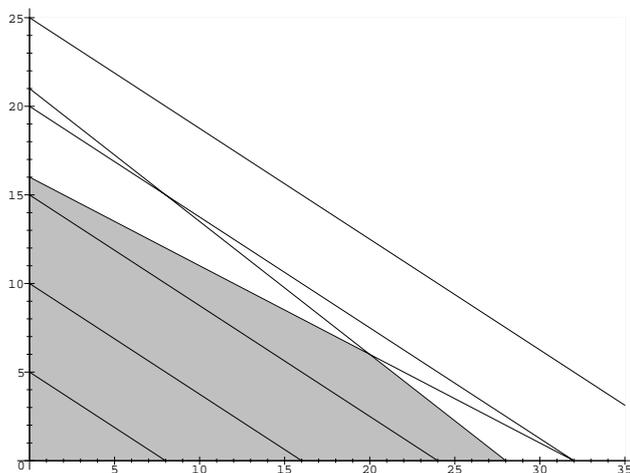

Figure 1: Feasible region and isoprofit lines for Example 1

The isoprofit lines are parallel to each other and are further from the origin for larger values of P. The optimal solution occurs at the corner point where the straight-line boundaries $x_1 + 2\,x_2 = 32$ and $3\,x_1 + 4\,x_2 = 84$ of the feasible

region intersect. This is because the isoprofit line through this point is the furthest such line from the origin that still intersects the feasible region. The optimal solution (20, 6) is found either by reading the coordinates of the corner point from the graph or by solving the equations $x_1 + 2\,x_2 = 32$ and $3\,x_1 + 4\,x_2 = 84$ simultaneously. The maximum profit is then determined by using the **subs** command to substitute the values for x_1 and x_2 in the expression for P.

```
>   solve({x[1]+2*x[2]=32,3*x[1]+4*x[2]=84},{x [1],x[2]});
```
$$\{x_2 = 6,\ x_1 = 20\}$$
```
>   subs(",P);
```
$$1480$$

The double quote symbol " here refers to the set of two equations resulting from the previously entered **solve** command. (Note that the order of the two equations in this set is immaterial and may change if the **solve** command is re-executed. On the other hand, the order of the coordinates specifying a point $(x_1,\ x_2)$ *is* significant.) The following graph confirms that the isoprofit line $P = 1480$ does pass through the corner point (20, 6) corresponding to the optimal solution.

```
>   inequal({x[1]+2*x[2]<=32,3*x[1]+4*x[2]<=84,x[1]>=0,x[2]>=0,
P=1480},x[1]=0..35,x[2]=0..25,optionsfeasible=(color=grey),opt
ionsexcluded=(color=white),optionsclosed=(color=black,thicknes
s=1));
```
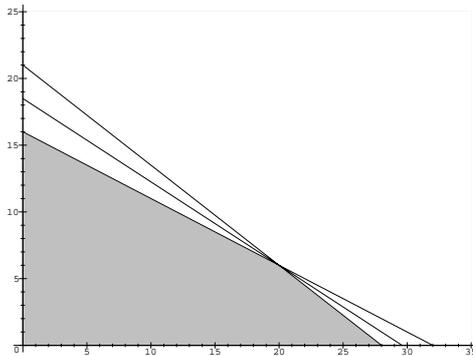

Figure 2: Feasible region and isoprofit line $P = 1480$ for Example 1

Theorem 1 on Page 279 of the textbook is the *Fundamental Theorem of Linear Programming*. It states that if the objective function does have an optimal value, then that value must occur at one (or more) of the corner points of the feasible region. (If the optimal value exists and occurs at two corner points, then it also occurs at every point on the straight-line boundary

joining those two points.) Theorem 2 on Page 279 gives three sets of conditions which can often (though not always) be used to determine whether an optimal value exists (that is, whether the linear programming problem does have a solution). We will make use of Theorems 1 and 2 in the following example.

Example 2

Find the maximum and minimum values of the objective function defined by $z = 3x_1 + x_2$ subject to the constraints $2x_1 + x_2 \leq 20$, $10x_1 + x_2 \geq 36$, $2x_1 + 5x_2 \geq 36$, $x_1 \geq 0$ and $x_2 \geq 0$.

This is Example 2A on Page 280 of the textbook. We first use the **inequal** command to obtain a graph of the feasible region.

```
>   inequal({2*x[1]+x[2]<=20,36<=10*x[1]+x[2],36<=2*x[1]+5*x[2],
0<=x[1],0<=x[2]},x[1]=0..10,x[2]=0..20,optionsfeasible=(color=
grey),optionsexcluded=(color=white),optionsclosed=(color=black,
thickness=1));
```
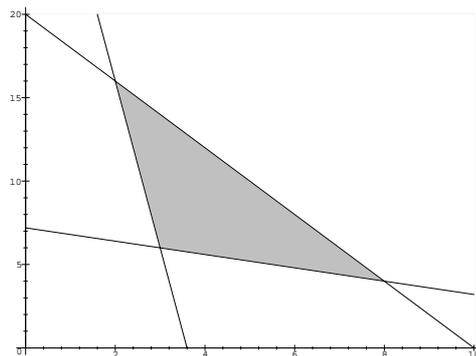

Figure 3: Feasible region for Example 2

It is clear from the graph that the feasible region (shown in grey) is bounded and it follows then from Part A of Theorem 2 that the objective function has both a maximum and a minimum value. By Theorem 1, these values occur at corner points of the feasible region. To determine the maximum and minimum values, we first find the coordinates of the three corner points, either by reading them from the graph or by solving the equations of the three straight-line boundaries in pairs, as follows.

```
>   c1:=solve({2*x[1]+x[2]=20,10*x[1]+x[2]=36},{x[1],x[2]});
```
$$c1 := \{x_1 = 2,\ x_2 = 16\}$$
```
>   c2:=solve({10*x[1]+x[2]=36,2*x[1]+5*x[2]=36},{x[1],x[2]});
```
$$c2 := \{x_1 = 3,\ x_2 = 6\}$$

```
>  c3:=solve({2*x[1]+5*x[2]=36,2*x[1]+x[2]=20},{x[1],x[2]});
```

$$c3 := \{x_1 = 8,\ x_2 = 4\}$$

We have assigned the results of the three **solve** commands that determine the corner points to the Maple variables c1, c2, and c3. If the objective function $3\,x_1 + x_2$ is now assigned to the variable z, then its value at each corner point can be determined by substitution.

```
>  z:=3*x[1]+x[2];
```

$$z := 3\,x_1 + x_2$$

```
>  subs(c1,z);
```

$$22$$

```
>  subs(c2,z);
```

$$15$$

```
>  subs(c3,z);
```

$$28$$

It follows that the minimum value of z is 15 and occurs at the corner point given by $(x_1,\ x_2) = (3,\ 6)$ (corresponding to c2) and that the maximum value is 28 and occurs at the corner point given by $(x_1,\ x_2) = (8,\ 4)$ (corresponding to c3).

A.4 Independent Problem Solving

Use Maple to solve problems 9, 15, 19 and 23 on Pages 285 and 286 of the textbook.

References

[1] R. A. Barnett and M. R. Ziegler. *College Mathematics.* Prentice-Hall, Upper Saddle River, NJ, seventh edition, 1996.

[2] G. F. Fitz-Gerald and W. P. Healy. Enlivening the Mathematics Curriculum with Maple. In Robert J. Lopez, editor, *Maple V: Mathematics and its Application*, Proceedings of the Maple Summer Workshop and Symposium, pages 71–80, Boston, August 1994. Rensselear Polytechnic Institute, Troy, NY, Birkhäuser.