Polynomial Algorithms for Optimization Problems

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Abstract

Optimization problems discussed as mathematical programming are expressed as

$$\begin{array}{ll} \text{minimize} & f(\boldsymbol{x}) \\ \text{subject to} & \left\{ \begin{array}{ll} g_j(\boldsymbol{x}) \leq 0 & (j = 1, 2, \ldots, m) \\ h_k(\boldsymbol{x}) = 0 & (k = 1, 2, \ldots, l) \end{array} \right. \end{array}$$

A problem which maximize the function is formulated similar to the above. Since the problem is solved by floating point computations in usual, there are some difficulties as below:

1. It is easy to obtain local minima. However, it is difficult to obtain global minima especially the case where several values of local minima are approximately same.

2. A computations for obtaining the local minima is strongly depend on its initial value.

Thus in this paper, the problem is solved algebraically. The objective function and constraints are changed to a set of polynomials equations by adding slack variables. The set of polynomial equations is solved by algebraic Gröbner basis and Sturm's theorem.

As further application of our algebraic computation, the same method is applied to geometrical problems such as

Find a circle which inscribes a given ellipse.

Thus subjects discussed in this paper are strongly connected to problems of geometrical theorem proving.