# Undergraduate students' analysis of problems within the MAPLE environment

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## Introduction

In recent times tertiary mathematics educators have taken an active interest in the utility and power of technology in promoting student interest in the subject matter as well as improving the performance of students in mathematics. This shift in instructional strategy can be attributed, among other things, to the increasing concern with the low participation levels shown by students. There is now an emerging consensus that technology could play a significant role in improving students' interest in the subject and the level of their engagement with mathematical concepts and procedures.

In this context an area of technology that has received considerable attention in tertiary mathematics involves computer algebra systems (CAS). Examples of CAS that has been investigated for use as effective tools to enhance instruction and student participation include Derive, Maple, Reduce and Mathematica. On the whole these apparent diverse systems have had some success with students' learning of mathematics and their ability to solve problems. While this is welcome news for the many enthusiastic advocates of CAS, the widespread use of these systems has raised a number of important questions about the efficacy of this class of instructional systems (Clements & Battista, 1994; Crane, 1991). An issue of general interest to educators and researchers alike concerns the mechanisms by which CAS promotes mathematical activity. More specifically, researchers are interested to understand the interaction between mathematical knowledge that students bring to the learning environment and facilities provided by CAS (Harper, Wooff & Hodgkinson, 1991; Kaput, 1986).

This is a report of a study that attempted to generate data relevant to the above issue. At the Curtin University of Technology we have been trialling several commercially available software packages including Maple. Maple is one of a number of currently used CAS at many universities in Australia.

## Prior mathematical knowledge use and technology

A general concern with mathematical performance has given rise to a stream of research that examines the role of prior knowledge when students attempt to acquire new knowledge or deploy prior knowledge during the search for solutions to problems. Results of these investigations suggests that the nature of prior knowledge that resides in the long-term memory could either enhance or impede the activation of that knowledge during mathematical task performance (Prawat, 1989). While this position seems to be a reasonable one, there are other factors that could have an equally influential role in the activation of prior mathematical knowledge. One such factor is the degree of knowledge elicitation that can be brought about when students work within a computer-based environment such as CAS.

An important requirement for the active use of prior knowledge is that students should be able to see the connection between information that is located in memory and that which is presented in the problem (Alexander & Judy, 1988). Thus, successful solution attempts involve the recognition of two components of knowledge: that which is stored in memory and that which resides in the problem. By drawing on the prior knowledge that is appropriate to making progress with the problem at hand, students are able to make effective use of previously-learnt knowledge in the search for the solution (Newell, 1990). This recognition of the relationship between the two above categories of knowledge could be achieved in two ways. Firstly, students could discern the connection between the knowledge components without external help, i.e., students spontaneous use of prior knowledge by establishing the required relations by themselves without any assistance. The second situation involves performances during which students are given some level of help to retrieve prior knowledge that is relevant to the solution. This assisted knowledge activation could take several forms including prompting, cueing and hinting.

Students who are able to use prior knowledge spontaneously would generally belong to the high-achieving group and those who experience difficulty with the activation of prior knowledge would most likely belong to the low-achieving group. This line of thinking suggests that most of our low-achieving students would show greater levels of knowledge activation and use if they are given help in the form of hints and prompts etc., and CAS seems to be rich in providing such useful clues to the students.

A cursory look at the characteristics of most CAS suggest that they could be utilised as tools of instruction in which the identification of hints or clues in the problem is facilitated. For instance, CAS could possibly play an influential role in situations where students are unable to recognise clues located in the problem environment. The clues that one could generate with the help of CAS are different and less direct than those that students receive from lecturers and tutors. Clues from the CAS are based on the multitudes of options and facilities that are available in the software. For example, if a student is working on a problem involving concepts in linear algebra, the awareness of the procedures available on the CAS for working out eigenvectors and eigenvalues rapidly would encourage a processing strategy in which students are encouraged to explore prior knowledge involving eigenvalues and eigenvectors. We are not suggesting that such help will lead to correct solution, only that the options in CAS would increase the likelihood of the activation and use of prior knowledge about vectors.

In this study we took up the above general issue of the instructional effect of CAS on students' mathematical knowledge access and use. Specifically, we wanted to examine the types of mathematical knowledge that students activated when they were instructed about how MAPLE could be utilised to generate graphical representations of a class of problems.

## Methodology

<u>Participants</u>

The two students involved in this study were chosen voluntarily from a standard calculus course for science students at the Curtin University of Technology. The participants were considered as above average ability on the basis of their performance on examinations conducted by the School of Mathematics and Statistics.

Procedure

A set of mathematical tasks involving concepts in calculus, analytical geometry and linear algebra was selected. These tasks were evaluated by a staff member at Curtin University who has had considerable experience in the use of MAPLE for learning and teaching tertiary mathematics. A primary consideration in the selection of the final task was the role that MAPLE (Ellis, Johnson, Lodi and Schwalbe, 1992) could play in helping students activate prior knowledge components that were relevant to the task. A second concern was the degree of mathematical knowledge integration that students needed to produce while they worked on the task.

In the final task, students were asked to read and execute a series of instructions that were relevant to generating geometric figures. These instructions provided directions about what MAPLE commands to use, and the rationale for using those commands. For example, students were informed that in order to draw geometrical figures they had to use commands such as 'with (linalg)' and 'with (plots)' to load the relevant packages. Students were also told that commands like 'polygonplot' could help them visualise geometrical figures better. The task required students execute the following five MAPLE commands: with(linalg), with(plot), p0, polygonplot3d, axes. All MAPLE and other instructions were presented on a sheet of paper.

Before the task was given to the students, they were allowed to go through the many options that were available on MAPLE. Both students have had about six months exposure to MAPLE. Students were given 10 minutes to read the task and discuss it among themselves. At the end of this session, students were asked to talk aloud as they attempted to complete the task. Students' work was video recorded and later transcribed for analysis.

## Results and Discussion

Consistent with the purpose of the study, data analysis focused on exchanges between the students that indicated activation of mathematical and MAPLE-related knowledge relevant to analysis and representation of the given problem. Hence our interest was in examining what prior knowledge was used by the students, and what role, if any, did MAPLE play in assisting students activate the knowledge. Table 1 shows the results of this analysis. Appendix 1 shows the MAPLE worksheet generated by the students.

Table 1: Knowledge activation with MAPLE

| **Solution Phase** | **Prior knowledge** | **MAPLE commands** |
|---|---|---|
| Problem Analysis | Definition of body diagonals<br>Trigonometry<br>Regular tetrahedron<br>Unit vectors<br>Axes<br>Problem categorisation<br>Graphing<br>Geometrical | >p0:=vector<br>> axes=boxed,orientation<br>>polygonplot3d<br>>p0:=vector |
| Problem Representation | Vector | > with(linalg):<br>> with(plots): |

Table 1 shows that during both the phases of problem analysis and problem representation students activated a number of important mathematical and related prior knowledge. Problem analysis required students to search their memory for definitions involving body diagonals. The given figure also prompted students to think about knowledge of trigonometry and two dimensional figures such as right triangles. While these were required to analyse the problem, it was knowledge about unit vectors and axes that led the students to think about visualising the figure more dynamically.

The students could have used pencil and paper to sketch the figure but MAPLE provided them with a facility to sketch such figures. This graphing facility of MAPLE has two significant advantages. Firstly, student were able to graph a function much faster than using the pencil and paper method. Secondly, the ease of sketching in MAPLE acts as an incentive to students to consider graph sketching in their repertoire of problem solving strategies, a point that was emphasised by Schoenfeld (1985) in his analysis of mathematical problem solving and the use of appropriate strategies.

The advantages conferred by MAPLE in promoting mathematical knowledge use could be given the following interpretation from a cognitive point of view. Sweller (1988, 1989) in his examination of mental resources involved in human problem solving demonstrated that in order for the solver to attend to the more novel and difficult parts of the problem in question, he must free his mental space. One way to achieve this is to reduce the cognitive load imposed by the less important tasks or aspects of the problem such as graph sketching, thereby permitting the solver focus his attention and mental resources on the more relevant and demanding part(s) of the problem. This line of argument about information processing during problem solving suggests that MAPLE, by allowing students to sketch a figure rapidly could have facilitated students' efforts in making progress with the problem. While making progress may in itself not be sufficient to the successful solution outcome, MAPLE does encourage knowledge searches which are more critical for that outcome.

During the second phase of the solution attempt students explored the problem further by constructing a representation of the problem using mathematical concepts and procedures from vectors, i.e., the given problem was interpreted via a vector representation. It is possible that students could have initiated the above move without the aid of MAPLE. However, we argue that the processing of the problem using vectors was encouraged by students' prior experience in drawing on this facility of MAPLE. Our past trials with MAPLE has showed that students tend to become familiar with some of the more commonly used procedures in MAPLE. One such procedure involves setting up vectors to examine a class of problems that involve three dimensional figures. Given that the target problem of the present study did contain a solid figure it appears that students' decision to reinterpret the problem from a new angle was promoted by MAPLE. Likewise, MAPLE could also foster students' patterns of reasoning that could lead them onto developing more sophisticated representations of the problem, an action that is supported by the knowledge rich computer environment (Kaput, 1987).

The above situation not only aided in the construction of an alternative representation, but also triggered a further chain of knowledge activation but this time students drew on and evaluated the usefulness of their prior knowledge of trigonometry and right-angled triangles, a process that was observed in a geometry problem-solving attempt by Lawson and Chinnappan (1994). In sum the transformation of the problem has helped students not only view the problem from another perspective but also promote the activation and use of further prior mathematical knowledge which would otherwise have remained dormant.

In the study reported here we were primarily concerned with helping students access, explore and apply previously-learnt mathematical knowledge that is relevant to a class of problems that require the establishment of graphical representation. The possible effects this cognitive activity on the quality of problem search with more novel situations needs to be investigated.

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## References

Alexander, P. & Judy, J. (1988). The interaction of domain-specific and strategic knowledge in academic performance. *Review of Educational Research*, *58*, 375-404.

Clements, H. D., & Battista, M. T. (1994). Computer environments for learning geometry. *Journal of Computing Research*, *10*, 2, 173-197.

Crane, M. (1991). The impact of MATLAB on teaching numerical mathematics. In B. Jaworski (Eds.), *Technology in Mathematics Teaching*, University of Birmingham, 181-188.

Ellis, W., Johnson, E., Lodi, E., & Schwalbe, D. (1992). *Maple V Flight Manual*. Pacific Grove, Calif: Brooks/Cole Publishing Company.

Harper, D., Wooff, C., & Hodgkinson, D. (1991). *A guide to computer algebra systems*. NY: Wiley.

Kaput, J. J. (1986). Information technology and mathematics: Opening new representational windows. *Journal of Mathematical Behaviour, 5*, 187-207.

Kaput, J. J. (1987). Multiple representations and reasoning with discrete intensive quantities in a computer-based environment, in *Proceedings of the Eleventh International Conference of Psychology of Mathematics Education*, Montreal, Vol 2, 289-295.

Lawson, M.J., & Chinnappan, M.(1994). Generative activity during geometry problem solving: Comparison of the performance of high-achieving and low-achieving students. *Cognition and Instruction*, 12 (1), 61-93.

Newell, A. (1990) Unified thoeries of cognition. Cambridge, Mass: Harvard Univeristy Press.

Prawat, R. (1989). Promoting access to knowledge, strategy and disposition in students. *Review of Educational Research*, *59*, 1-42.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. New York: Academic Press.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, *12*, 257-286.

Sweller, J. (1989). Cognitive technology: Some procedures for facilitating learning, problem solving in mathematics and science. *Journal of Educational Psychology*, *81*, 457-466.

**Appendix 1**

**MAPLE Worksheet**


```
{VERSION 2 3 "IBM INTEL NT" "2.3" }
{USTYLETAB {CSTYLE "Maple Input" -1 0 "Courier" 0 1 255 0 0 1 0 1 0 0
1 0 0 0 0 }{PSTYLE "Normal" -1 0 1 {CSTYLE "" -1 -1 "" 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 }0 0 0 -1 -1 -1 0 0 0 0 0 0 -1 0 }{PSTYLE "Maple Plot
" 0 13 1 {CSTYLE "" -1 -1 "" 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 }3 0 0 -1
-1 -1 0 0 0 0 0 0 -1 0 }{PSTYLE "" 0 256 1 {CSTYLE "" -1 -1 "" 0 1 0
0 0 0 0 1 0 0 0 0 0 0 0 0 }3 0 0 -1 -1 -1 0 0 0 0 0 0 -1 0 }{PSTYLE ""
0 257 1 {CSTYLE "" -1 -1 "" 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 }3 0 0 -1
-1 -1 0 0 0 0 0 0 -1 0 }}
{SECT 0 {EXCHG {PARA 256 "" 0 "" {TEXT -1 10 "Appendix 1" }}{PARA 0 "
" 0 "" {TEXT -1 0 "" }}{PARA 257 "" 0 "" {TEXT -1 15 "MAPLE Worksheet
" }}{PARA 0 "" 0 "" {TEXT -1 0 "" }}{PARA 0 "" 0 "" {TEXT -1 0 "" }}
{PARA 0 "" 0 "" {TEXT -1 0 "" }}{PARA 0 "" 0 "" {TEXT -1 0 "" }}}
{EXCHG {PARA 0 "> " 0 "" {MPLTEXT 1 0 13 "with(linalg):" }}}{EXCHG
{PARA 0 "> " 0 "" {MPLTEXT 1 0 12 "with(plots):" }}}{EXCHG {PARA 0 "> \+
" 0 "" {MPLTEXT 1 0 48 "p0:=[0,0,0]:p1:=[1,1,0]:p2:=[1,0,1]:p3:=[0,1,1
]:" }}}{EXCHG {PARA 0 "> " 0 "" {MPLTEXT 1 0 60 "polygonplot3d([[p0,p1
,p2],[p0,p1,p3],[p2,p3,p1],[p2,p3,p0]]," }}{PARA 0 "> " 0 "" {MPLTEXT
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1 0 35 "axes=boxed,orientation = [145,70]);" }}{PARA 13 "" 1 ""
{INLPLOT "6%-%)POLYGONSG6&7%7%\"\"!F(F(7%$\"\"\"F(F*F(7%F*F(F*7%F'F)7%
F(F*F*7%F,F.F)7%F,F.F'-%*AXESSTYLEG6#%$BOXG-%+PROJECTIONG6%$\"$X\"F($
\"#qF(F+" 3 303 303 303 1 0 1 0 2 1 0 2 2 1.000000 70.000000
145.000000 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 -30663 0 0 0 0
0 0 1 }}}{EXCHG {PARA 0 "> " 0 "" {MPLTEXT 1 0 0 "" }}}}{MARK "0 4 0"
0 }{VIEWOPTS 1 1 0 1 1 1803 }