Connecting Mathematics with Machine Engineering and Art: Perspectives for Calculus and Geometry for All via Technology

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Abstract

The paper illustrates some perspectives for curriculum reform via using technology in the East Asian countries where teaching Geometry in junior high schools and Calculus in senior high schools and have to change their curriculum more constructive one. Exploring Family of Function changes connection between Algebra and Calculus more meaningfully than Pre-Linear Algebra and Pre-Calculus. Exploring linkage connect Geometric Construction with Geometry, Algebraic Geometry, and Engineering. In these explorations, Art is a new area for appreciation and applying of mathematics.

1 Using Technology in the Asian Settings

In the last twenty years, mathematicians have been using technology for research. Horgan., J. [3] says it is an innovation of mathematics research described by a word ‘The Death of Proof’. The topic is sensational for secondary school mathematics teachers but what concretely means by that is not sensational. We should discuss what is innovation for secondary mathematics as reform via technology in our (Asian) settings.

The result of *Third International Mathematics and Science Study* [10] shows that some of East Asian countries’ (and Hong Kong’s) students scored higher than other countries. Especially, Korea and Japan have the same features that they do not use computer for learning/teaching mathematics but they achieved high. Some other countries' teachers believe that to use computational tools is necessary for supporting lower achieved students' skill. But in case of Japan and just in case of high school teachers, they usually believes that they should teach skills to lower as same as higher achieved students. This belief is one of the reason to account for why Japanese students could get such higher achievement. This belief is shared among East Asian countries. This was influenced by Chinese culture. For example, the belief may be originated from using Chinese abacus, our traditional calculator. To use a Chinese abacus, we have to do mental calculation on addition, subtraction, multiplication and even division. In the past, to learn mathematics meant to master abacus's operation skillfully. And in case of Japan, This belief may be one of the reasons why students feel difficulty in learning mathematics and them become hating it. In fact, teachers know the situation but they tend to ignore them because doing such way enable students to take higher achievement. Thus, for East Asian countries, instead of enable students to get higher achievement, we need more persuasive ways to show the power of using technology.

When we see the power of technology for mathematics learning, we believe that the laboratory approach [8] and Visualization [19] can enhance student's learning and change learning situations for exploring and hopefully inquiring [14]. In the case of the textbook CALCULUS Graphical, Numerical, Algebraic written by Finney, R., Thomas, G., Demana, F. and Waits, B. [2], the changes are summarized as the following methods:

- Do a problem algebraically, then support the results numerical and/or graphically.
- Do a problem numerically and/or graphically them confirm the result algebraically.
- Do a problem numerically and/or graphically because paper and pencil methods are impractical.

Even it is difficult for student to construct mathematics, every teacher wants them to do. However teachers' expecting activity is usually based on paper and pencil approach. Showing teachers with nice examples by which each students can explore a problem, discuss with other and appreciated with the exploration would be the most persuasive way to change his/her belief. In this context, we should know that the main problem for us is what should be taught. If we want to teach some topics and regarding that previous methods are appropriate for the students to perform activity on them, we might use technology instead. With the power of technology, it is not difficult to share the previous method with teachers all over the world thorough persuasive examples. However the contents which we could apply these methods may be more difficult to be shared because what should be taught would change dramatically. Even though this problem is not only for Asian countries but we prefer to discuss it in Asian settings.

Each Asian country has its own settings. For countries which had been influenced by Chinese culture might have some similar settings. We have been growing up through radical industrialization. Although the development and manufacturing technology support this growing, in case of Korea and Japan, we rarely use it for education. Most powerful motivations driving students to learn mathematics is to pass an entrance examination. It is because of this that many students hate mathematics. We have a cram school. Comparing our textbooks with US and some other countries, we know that we rarely show students that the mathematics is useful for other subjects and mathematics can be applicable. We have national curriculum. We teach students the proof on Euclidean Geometry in the lower secondary level before Algebraic Geometry (Analytic Geometry) and Pre-Calculus but they could not use it after they learned it. We usually teach Calculus in the upper secondary school without computer. In the case of US and some English speaking countries, Euclidean proof and Calculus are usually prepared as the optional courses in the upper secondary level. To use computer is regarded as necessary for enhancing students' learning of Calculus or even Algebra.

If we reflect on our own settings, we should reform our curriculum to become, more appreciable and applicable. We should not lower down our curriculum level in doing so. Calculus and Geometry should be taught for almost students because these contents are necessary for industrial countries and we have been already teaching them without computer. But the contents of Calculus and Geometry should be changed to become more appreciable and applicable by using technology. Then technology would be a helpful tool for our reform because it would be persuasive for many secondary mathematics teachers. This is the reason why I want to connect mathematics with machine engineering, art, and mathematics itself via technology. I hope that students could connect mathematics with other subjects and itself, and appreciate their own activity depending on their own beliefs and change their own beliefs for a more constructive one.

For our curriculum reform, I want to discuss three perspectives in this paper.

- Exploring the Family of Function could change connection between Algebra and Calculus more meaningfully than Pre-Linear Algebra and Pre-Calculus.
- Exploring the linkage connect Geometric Construction with Geometry, Algebraic Geometry, Calculus and Machine Engineering.
- In these explorations, Art is a new area for appreciation and applying of mathematics.

Next, I will illustrate some examples about the family of function and the linkage.

2 Exploring the Family of Function

To explore the family of function is one of the typical ways for using graphing utility [11, 18]. In the case of entire function, if we could solve an equation algebraically, we could get the tangent, extremal value and inflection point of it without using calculus [6, 11, 16]. For example, in the case of $f(x) = ax^2 + bx + c$, students could guess the tangent line on $(0, f(0))$ is $y = bx+c$ from an family of $f(x)$ such as Figure 1. They confirm it algebraically and generalize it on $(X, f(X))$ and more higher degree of entire function. After that, when they learn differentiation, they know this method is the expression of Taylor expansion on entire function.

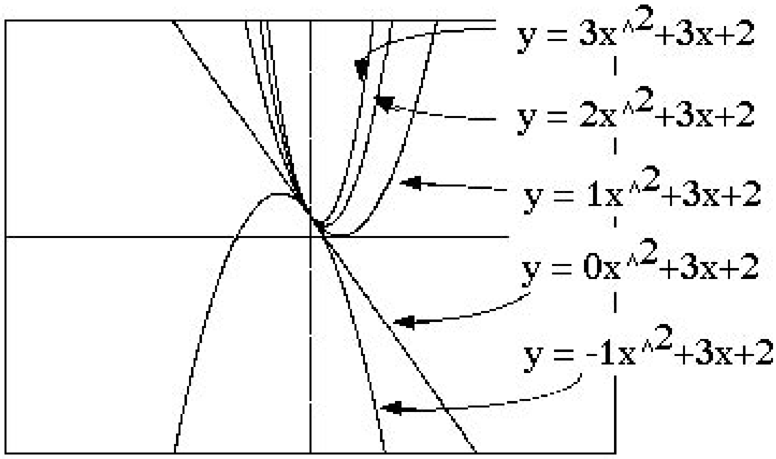

Figure 1: Family of Parabola and tangent line on (0, 1).

Before graphing utility, to learn function means to know some features of it, typically to know graph in Pre-calculus. The first application of differentiation is to draw graph using differentiation because to know the shape of graph strictly is not easy in Pre-calculus. After we use graphing utility, we could explore function as family instead of merely drawing graphs by using paper and pencil method. If we use graphing utility, we could spread the applicability of algebra and differentiation would be learned more constructively based on such experiences. It is needless to show examples how students could inquire family of function after they learned differentiation.

In the family of function, we could find another new situation for exploring functions on the context of drawing. Figure 2 is done by a student only using parabola [15]. It looks fun but suggesting some idea about the family of it. To make the family, the idea of transformation is useful. Figure 3 is linear transformation of Cat [5]. ¿From the changing of Cat, we could know the feature of some family of linear transformation. This is another generalization about the family of function including the idea of drawing. Figure 4 is another transitional example from the family to the idea of matrix. Solution of simultaneous linear equations $x - 2y = -1$, i.e. $2(y - 1) = x - 1$ and $3x - y = 2$, i.e. $y - 1 = 3(x - 1)$ is $(1, 1)$. This is not only visualization but the method of solution by addition and subtraction of simultaneous equation would be generalized to matrix, i.e. linear algebra.. In fact, any equation such as $a(x - 2y) + b(3x - y) = a(-1) + b(2)$, i.e. $(2a + b)(y - 1) = (a + 3b)(x - 1)$, when a and b are not equal zero at the same time, go through $(1, 1)$.

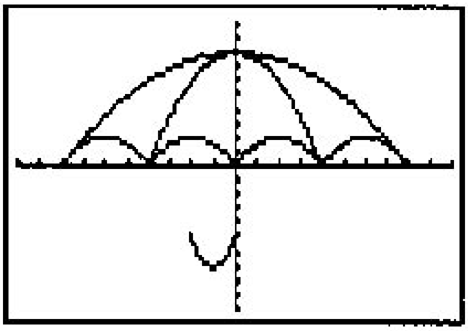

Figure 2: A Umbrella [15].

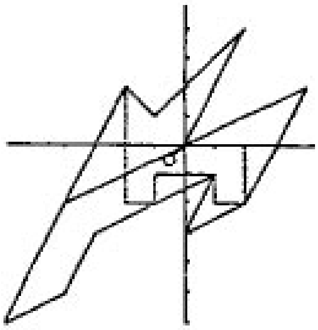

Figure 3: Cats [5].

The sense about the family of function is useful not only in Mathematics but also in Art. Figure 5 is drawn by Sasaki, M. [12] using Computer-art systems ART (Artistic Realistic Technician) developed by Sasaki, T. who is also well known in approximate algebra in computer algebra area. The ART generates the famly of function. His family have been using it for drawing their computer art. For drawing, we need the sense about the family of function. We should know that this sense is needed to apply function not only in Art but also more in the other areas.

The idea of the family of function with using graphing utility expands the situation which would be applied for algebra and differentiation. Through the inquiry of the situation, we could compare the method of algebra and differentiation and we could appreciate them. The idea is applicable to the transformation and Art. Via the inquiry of the situation, we could connect mathematics with Art and mathematics itself more usefully.

3 Exploring Linkage

In some East Asian countries, we teach Euclidean Geometry before Algebraic Geometry in the junior high schools and teach Algebraic Geometry before Calculus in

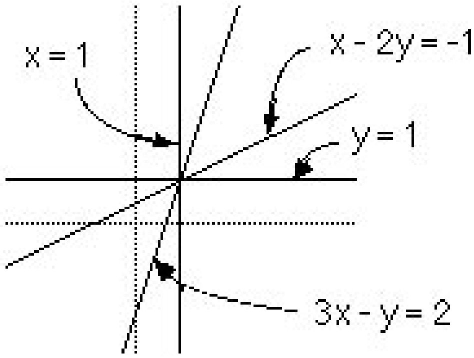

Figure 4: Solution of Equations.

Figure 5: Mt. Fuji with Tornado [12].

the senior high schools. To know how and why to draw a curve using linkages, we have to know Euclidean Geometry and Algebraic Geometry. For example, Conic curves are beautiful, harmonized nature on construction via linkages as Figure 6, 7 and 8. If we change the directrix of parabola in Figure 6 to circle, we get ellipse like Figure 7. If we move the focus of ellipse which is one hand of foci and focus of parabola in Figure 6 to the outside of circle in Figure 7, we get hyperbola like Figure 8. If we use construction tool like Cabri or Geometer's Sketchpad, we could draw below Conic curves with the same way. From this construction, we could know harmonized nature of Conic curves.

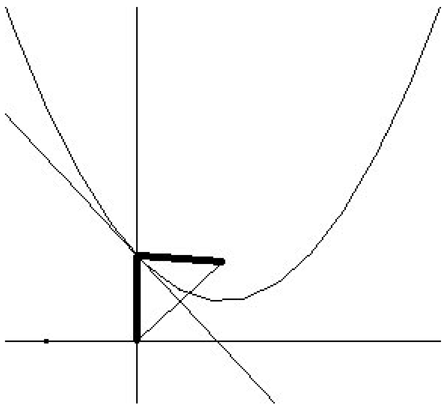

Figure 6: Parabola.

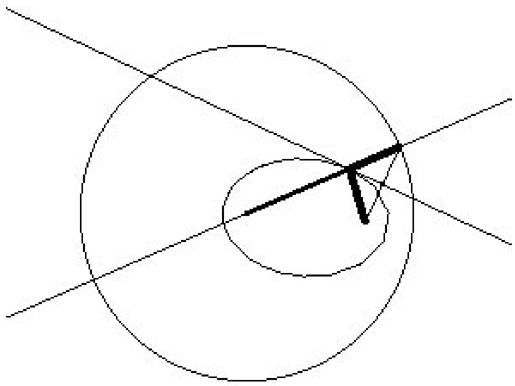

Figure 7: Ellipse.

This drawing shows that one of line for constructing locus is a tangent of each curve in these cases. If we use construct locus with technology, we could easy know each curve has drawn by its envelop, i.e. tangent. Students who know Calculus could see the envelop from the view point of Calculus. If you know paper

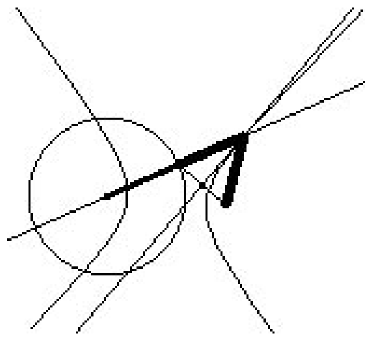

Figure 8: Hyperbola.

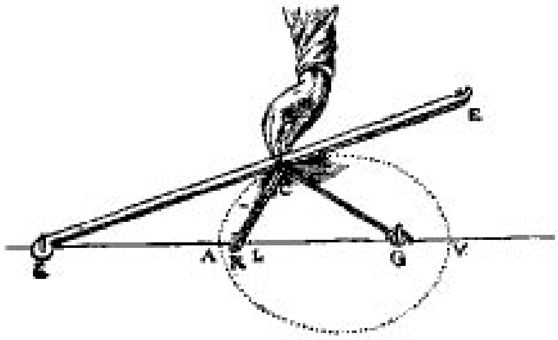

Figure 9: Geometry [1].

folding/origami, Japanese traditional art, well, you could fold the envelop using only a piece of paper [4]. Thus, these constructions connect among Geometry, Algebraic Geometry, Calculus and even Japanese traditional art.

Figure 9 is one of the pictures from the Geometry written by Descartes, R. [1]. He discussed that it is obscure for him that the ancients wanted to discuss geometric construction only using circle and line and distinguish it from mechanical construction like Figure 9 or so because to draw circles and lines we need some tools. He said that the ancients would have been their own intuition which he could not understand. He needed new intuition which easier than the ancients' one. Thus, he developed Algebra and Algebraic Geometry as new analysis.

In case of Japan, curves drawn by linkages were the basics for secondary school mathematics before the occupation of the allied nations (or USA) after World War II. Japanese integral method problem about some figures by Linkages using circles had been already popular in the early 1800's in Japanese mathematics textbook. The illustrations of Figure 10 are picked out from the mathematics textbooks for secondary school from 7th (twelve years old) to 10th grades published in 1943 [9]. The textbooks show that Linkages were discussed with the initiation of Geometry, Algebraic Geometry, Physics and Machine Engineering. The mathematics curriculum was integrated in order to develop the ability of mathematization and linked with physics and mechanics.

During the occupation period, Japanese school system was changed and mathematics curriculum was converted by progressivism. After the occupation, mathematics education faced to modernization. In these processes, linkages and locus had been eliminated from the curriculum because it is difficult to image the curve or movement of linkages on Euclidean Geometry, we could discuss it easier by using the method of algebra intended by Descartes and we should teach new idea of mathematics like linear algebra instead of it. So many mathematics teacher come to believe that mathematical idea could be constructed simply by itself. The belief separate mathematics from other subjects. Computer programming enhances the power of algebra and algebraic geometry, too. Nowadays, almost teachers could not see linkages as the important object for mathematics subject because they learn mathematics after World War II.

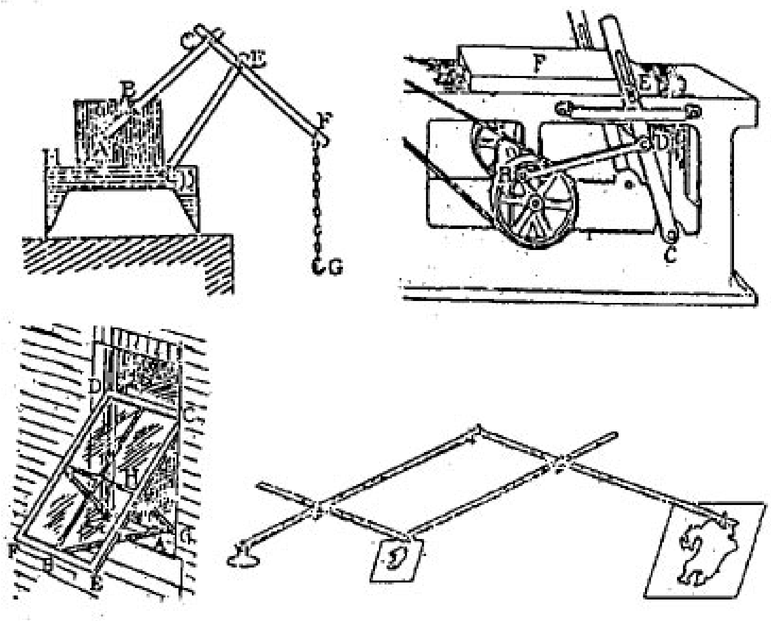

Figure 10: Linkages for Machine and Pantograph for drawing from Secondary Mathematics Textbook [9].

Geometric Construction tools on computer have changed this situation. They enable us to do the following situations for student's explorations just similar to graphical, numerical, algebraic methods. Do a problem constructively and/or using linkages to confirm the result geometrically and/or algebraically. Do a problem geometrically and/or algebraically, then support the results constructively and/or using linkages. Do a problem constructively and/or using linkages because paper and pencil methods are impractical (for just in case students). Because we teach geometry and geometric proof in junior high schools and teach Algebraic Geometry and Calculus in senior high schools, we could use these approach in high school. Before using Geometric construction tools on computer, we could only use linkages in these methods. But to do so, we have to make a linkage which is mechanics by teacher themselves varying case by case. It is not expecting activity for mathematics teacher who loves desk work. For example, thirty years ago, Pantograph created by Scheiner (1573–1650) in Figure 10 was a popular tool for high school teachers. This is an important tool for realistic art. We could see the activity of

Durer (1471–1528) as an origin of projection geometry and transformation geometry. Nowadays, it is difficult to find teachers who use pantograph in Japanese schools. Even one could find it in store-box, he/she does not know what is it and how to use it. Geometric construction tools on computer have changed this situation. By using technology, we could re-invent the function of traditional tools for mathematics. LEGO enables us to make mechanics of linkage easily as in Figure 11 [7]. Geometric construction tools enable us to connect Geometry with Algebraic Geometry, Calculus, Machine Engineering, and Art once more.

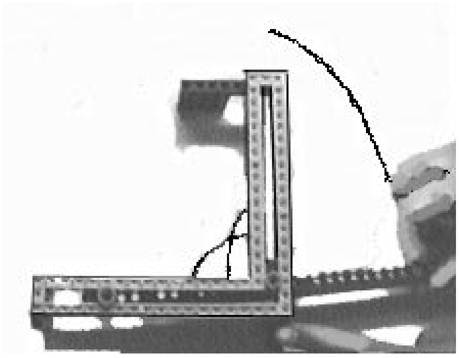

Figure 11: Linkage by LEGO.

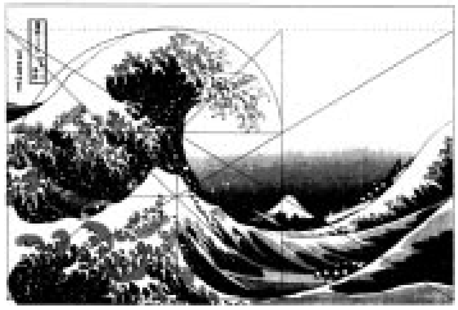

Figure 12: Mt. Fuji by Hokusai (Lines shows golden ratio by Yanagi [17]).

4 Final Remark

When I saw the art of Sasaki,M., it remind me about the color woodprint named UKIYOE like Figure 12 which had influenced Japonism in art in the latter half of the nineteenth century. If I say that there are several different kind of mathematics in the world, many mathematician and mathematics teachers would feel it funny. But until early of the ninteenth century, it was true. In case of Japan, mathematics (Japanese mathematics) researchers in the seventeen century were the top of the world in approximate calculation area on calculus. I do not want to say mathematics curriculum should be the same as history but I want to say that the mathematics curriculum in 1

Examples of family of function and linkage give our students with very nice situations that they can explore and hopefully inquire. Via exploration of these topics with technology, we could connect mathematics with other subject, Machine Engineering and Art as illustrated in this paper, and within mathematics itself. Technology enables us to explore these situations more easier than without it. In the field of Education, mathematics content itself will not change radically but the value of contents could be changed depending on what should be taught.

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